July 3, 2002

UCLA Department of Economics

First-Year Core Comprehensive Examination in MACROECONOMICS

Spring 2002

Answer only **FIVE** of the six questions offered. Each question has equal weight.

Answer each question in a separate answer book. Put the question number on the outside of each book.

You have **4 hours**

Question 1: Investment Under Uncertainty

Consider an industry with N (a constant) identical competitive firms that produce widgets according to a production function that is linear in capital (k_t). In particular, the quantity of output produced by a firm is $f\,k_t$, where f is a constant greater than zero. Industry output is $f\,K_t$, where K_t is the total stock of capital employed in the industry ($K_t = N\,k_t$ in equilibrium). The price of widgets is given by p_t, which is taken as exogenous by the firms. Industry demand is given by $p_t = a - b\,f\,K_t + u_t$. The term u_t is a mean 0 i.i.d. random variable realized at the beginning of period t.

Each firm comes into period t with k_{t-1} units of capital (capital does not depreciate) and chooses the amount of capital (and hence output) that it wishes to employ in that period (k_t). The price of capital is given by J_t, where $J_{t+1} = \gamma J_t + \varepsilon_{t+1}$ and $0 < \gamma < 1$. The term ε_t is a mean 0 i.i.d. random variable. The firms also face adjustment costs associated with changing the quantity of capital employed. This cost is equal to $\frac{d}{2}(k_t - k_{t-1})^2$.

Firms discount future profits at the rate β, where $0 < \beta < 1$. Hence, a firm chooses its production in order to maximize:

$$E\sum_{t=0}^{\infty}\beta^t\{p_t f\,k_t - J_t(k_t - k_{t-1}) - \frac{d}{2}(k_t - k_{t-1})^2\}$$, where p_t is determined according to the industry demand equation given above.

A. Derive Bellman's equation for the firms' optimization problem. Be careful to make sure that your dynamic program is completely specified.

B. Define a *recursive equilibrium* for this industry. Hint: This is analogous to a *recursive competitive equilibrium* in a dynamic general equilibrium model except that some prices (the price of capital) and behaviors (the demand for widgets) are taken exogenously.

C. Using the first order condition for the firm's problem and the equilibrium conditions, obtain a second order difference equation (Euler equation) in K that must hold in equilibrium.

D. Show how one can solve for the equilibrium law of motion for K_t using the method of undetermined coefficients. You only need to characterize the solution—you do not need to do the algebra.

Question 2: .Consider an economy with a population of a representative infinitely lived households with preferences given by,

$$\sum_{t=0}^{\infty} \beta^t N_t \log c_t .$$

Here, N_t is the size of the household in period *t*. The population is assumed to grow with $N_{t+1} = (1+\eta)N_t$, where $\eta > 0$ and $N_0 = 1$. Each household member is endowed with one unit of labor each period.

The technology is given by:

$$Y_t = \gamma^t K_t^{\theta} N_t^{\phi} L_t^{1-\theta-\phi}$$

Here, $\gamma > 1$ is the exogenous total factor productivity growth factor, K_t is total (not *per capita*) capital, Y_t is total output, and L_t is the total stock of land. Land is assumed to be a fixed factor; it cannot be produced and does not depreciate. To simplify with out loss of generality, assume that $L_t = 1$ for all *t*.

The resource constraint, assuming 100 percent depreciation of capital each period, is given by

$$N_t c_t + K_{t+1} \le Y_t .$$

A) Formulate, as a dynamic programming problem, the social planner's problem for this economy.

B) Characterize the balanced growth path. Solve explicitly for the growth rate of per capita consumption (c_t) along this path.

C) Now decentralize this economy by defining a *recursive competitive equilibrium.* Include markets for output, labor, capital services, land, and land services.

Question 3: Consider the following economy. There are a unit measure of each of two types of agents, whom we will label capitalists and workers.

Workers: are the only ones who can supply labor, and have preferences given by

$$\sum_{t=0}^{\infty} \delta^t \left[u(c_t) + v(1 - l_t)\right],$$

where c and l denote consumption and labor effort, both u and v are concave, $u'(0) = v'(0) = \infty$, and $\lim_{c \to \infty} u'(c) = v'(1) = 0$. Their flow budget constraint is

$$r_t k_{t-1}^w + w_t l_t + m_{t-1}^w \geq m_t^w + p_t c_t^w + p_t k_t^w,$$

where k_t^w and m_t^w denotes the holdings by workers (w) at the end of period t of capital and money respectively. Workers also face a liquidity restriction that consumption and capital cannot be purchased from labor income directly, or

$$r_t k_{t-1}^w + m_{t-1}^w \geq p_t c_t^w + p_t k_t^w. \tag{1}$$

Capitalists: Only derive their income from capital, and have preferences over consumption given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

and their flow budget constraint is

$$r_t k_{t-1}^c + m_{t-1}^c \geq m_t^c + p_t c_t^c + p_t k_t^c,$$

where the superscript "c" indicates a capitalist choice variable.

Key Assumption 1: We will assume that capitalists are more patient than workers

$$1 > \beta > \delta > 0$$

Key Assumption 2: Capital holdings and the money holdings must be nonnegative; i.e. $m_t^i, k_t^i \geq 0$ for $i = w, c$.

Technology: We will assume that there is an aggregate production technology $F(K_t, L_t)$, which is concave, $F'(0, L) = F(K, 0) = \infty$ for any positive L and K, and which has the property that $\lim_{K \to \infty} F(K, 1)$ is finite. The resource constraint for this economy is given by

$$F(k_{t-1}^c + k_{t-1}^w, l_t) = c_t^c + c_t^w + k_t^c + k_t^w.$$

Part a: Write out the capitalist's problem and derive their f.o.c.'s. Assume that we're in a steady state, and that in particular r_t and p_t are constant. Show that if the return on capital is given by $r_t/p_{t-1} = 1/\delta$ for all t, then the capitalists will accumulate more capital in every period, but will hold a constant nonzero amount if $r_t/p_{t-1} = 1/\beta$. Show that if $r_t/p_{t-1} > 1$ and the price level is constant, $p_t = p$ for all t, then capitalists hold zero money balances.

Part b: Write out the worker's problem and derive their f.o.c.'s. Again, assume that we're in a steady state. Show that if $r_t/p_t = 1/\beta$ for all t, then workers will hold zero capital. If workers are holding zero units of capital, what does this imply about their liquidity restriction (1) and the extent to which the pattern of their transactions is similar to what would arise under a cash-in-advance constraint?

Part c: Characterize the steady state of this economy.

Question 4: Consider the following economy. Assume there is no capital and that output is produced solely with labor. The realized output level depends upon the productivity shock z. We will also assume that government spending, g, is stochastic as well. We will assume that the only source of government revenue is a sales tax on consumption.

Let s^t which will denote the state in period t be given by the history of shocks, or

$$s^t = \{z_j, g_j\}_{j=0}^t.$$

We will assume that the initial state s^0 is not stochastic. Let $\mu(s^t)$ denote the probability of s^t. The resource constraint for this economy is given by

$$c(s^t) + g(s^t) = z(s^t)l(s^t).$$

The government's budget constraint is given by

$$g(s^t) + R(s^t)b(s^{t-1}) \leq \tau(s^t)c(s^t) + b(s^t),$$

where R denote the gross return on government debt.

The consumer's problem can be written as

$$\max \sum_t \sum_{s^t} \beta^t u(c(s^t), l(s^t))\mu(s^t)$$

subject to

$$(1 + \tau(s^t))c(s^t) + b(s^t) \leq w(s^t)l(s^t) + R(s^t)b(s^{t-1})$$

The firm's problem is given by

$$\max\ z(s^t)f(l(s^t)) - w(s^t)l(s^t) \text{ for each } s^t$$

Part a: Use the firm's and the household's foc's to characterize what prices R and w, and taxes τ must be in equilibrium as a function of the quantity variables. Then, assuming these relationships held, derive an implementability condition from the household's budget constraint.

Part b: Set up the Ramsey problem for the optimal choice as to the consumption tax policy, and use it to characterize the efficient allocation. Show that this implies that this allocation is a stationary function of the prevailing state s_t - i.e. it is history independent.

Part c: Could the government do better if it had labor taxes instead of consumption taxes?

Question 5: Consider an economy in which the households start with an endowment $k_t(h_t)$ which they can store from one period to the next no depreciation. In it, the household has preferences

$$\sum_{t=0}^{\infty} \beta^t \sum_{h_t} u(c_t(h_t))\eta_t(h_t)$$

where $\eta_t(h^t)$ denotes the probability of event h^t being realized at date t. The resource constraint for this economy is given by

$$c_t(h^t) + k_t(h^t) = k_{t-1}(h^{t-1}).$$

Now assume that at date zero one household wishes to sell an asset which represents a claim to a sequence of random dividends $\{y_t(h^t)\}_{t=0}^{\infty}$ where $y_t(h^t)$ is the dividend paid at date t if event h^t is realized at t. What is the price of this asset? Does your answer depend on the risk aversion of the agents in this economy?

Question 6: A Linear-Quadratic Growth Model:
Preferences for a representative consumer are given by:

$$\max E \sum_{t=0}^{\infty} \beta^t (c_t - \phi_t \frac{l_t^2}{2})$$

The resource constraint is:

$$Ak_t^\theta l^{1-\theta} + (1-\delta)k_t = c_t + k_{t+1}$$

Investment must be non-negative:

$$k_{t+1} \geq (1-\delta)k_t$$

The parameter ϕ is a random variable with strictly positive support. How do you interpret this parameter?

Can you define a recursive competitive equilibrium? If so, characterize the equilibrium. Now, suppose that the parameter ϕ is constant. Does this economy have a steady state? If so, solve for the steady state capital stock. What happens in this economy if the initial capital stock is below the steady state level? What happens if it is above the steady state level? How might your answers change if investment could be negative?

Now, suppose all aspects of the model remain as above with a fixed ϕ, but there is long-run productivity growth:

$$\begin{aligned} y_t &= Ak_t^\theta (x_t l_t)^{1-\theta} \\ x_t &= (1+\gamma)^t \end{aligned}$$

Does this economy have a steady state growth path?