QUALIFYING EXAM IN MACROECONOMICS

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Instructions: You have four hours to complete this exam. There are three parts. Each has equal weight. **Answer each part in a separate blue book.**

Part One (*Blue Book Number 1*)

Consider an economy in which the final consumption good is produced using a two-stage process that takes two periods to complete. In the first stage of production, land and labor are combined to produce an intermediate good, q, according to the constant returns to scale technology, $q_t = F^1(l_t, h_{1t})$. Here, l_t denotes the quantity of land and h_{1t} denotes labor assigned to the intermediate good sector. In the second stage, consumption goods are produced from labor and intermediate goods from the previous period using a constant returns technology, $c_t = F^2(q_{t-1}, h_{2t})$.

In addition, there is an infinitely lived household that maximizes utility given by $\sum_{t=0}^{\infty} \beta^t U(c_t, 1-h_t)$, where $h_t = h_{1t} + h_{2t}$. The household is endowed with one unit of land. Land cannot be created or destroyed.

A. Formulate the dynamic programming problem that would be solved a social planner in this economy. Derive the first order and envelope conditions that characterize an optimum.

B. Consider now a decentralized version of this economy with two profit maximizing firms, one producing intermediate goods and the other producing final goods, both of which are sold to the household. In addition, the firms hire labor, rent land, and purchase productive intermediate goods from the households. There are markets for final goods, new intermediate goods, productive intermediate goods, labor, and land (both resale and rental markets). Carefully define a *recursive competitive equilibrium* for such an economy.

C. Derive expressions for the steady state price of land and the price of new intermediate goods in terms of the parameters of the model, the rental price of land, and the price of productive intermediate goods. Provide intuition for your results.

D. Suppose now that the production of intermediate goods depends on physical capital rather than land (land is no longer a factor of production). Suppose that capital depreciates at the rate δ, takes one period to become productive, and is produced using the same technology as consumption goods. Carefully define a *recursive competitive equilibrium* for this economy.

Part Two (*Blue Book Number 2*)

(1) Consider a model with an infinitely lived representative agent, with utility

$$\sum_{t=1}^{\infty} \delta^t \log c_t$$

where $\delta \in (0,1)$. Technology is given by

$$y_t = A k_t^{1/2} h_t^{1/2}$$

where k is physical and h is human capital and A is a positive parameter. Feasibility requires

$$c_t + k_{t+1} + h_{t+1} \leq y_t + (1-\mu)k_t + (1-\mu)h_t$$

and the non-negativity of c, k, h at all dates. The depreciation rate $\mu \in (0,1)$. At time zero the representative agent is endowed with k_0 and h_0 units of physical and human capital respectively. Consider the problem of maximizing the representative agent's utility subject to the resource constraints.

- a) What restrictions on δ, A and μ ensure a positive balanced growth path for this economy?
- b) For what restrictions on δ, A and μ is the growth rate equal to zero, asymptotically?

(2) Consider the following economy. There is a single representative agent with objective function

$$\sum_{t=1}^{\infty} \delta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

where $\delta \in (0,1)$ and $0 < \sigma < \infty$. Once again the agent has k_0 units of physical and h_0 units of human capital. Moreover the agent is also endowed with one unit of time, which he can allocate between the production of the physical good and the accumulation of human capital through a privately owned technology. The technological possibilities of the economy are given by the following production function

$$y_t = Ak_t^{\alpha}(u_t h_t)^{1-\alpha}$$

where k is physical, h is human capital, u is time devoted to production, A is a positive parameter and $\alpha \in (0,1)$. Output can be devoted either to consumption c_t or investment i_t. Physical capital is determined by the law of motion

$$k_{t+1} = i_t + (1-\mu)k_t$$

while human capital is determined by

$$h_{t+1} = B(1-u_t)h_t + (1-\mu)h_t$$

with $B > 0$.

- a) Specify a competitive equilibrium for this economy.
- b) Characterize the steady state or balanced growth path for this economy.
- c) Explain how the steady state or balanced growth path would be changed if the economy opens up to trade, and faces a constant rate of interest r in the international capital market.

Part Three (*Blue Book Number 3*)

I. Consider an OLG economy with constant population (one individual per generation) where agents live for two periods and there is a single perishable consumption good. Agents are endowed with w units of the consumption good when young and they have zero endowment when old. The following is their utility function:

$$U(c_t, c_{t+1}) = \log c_{t+1} + \beta \log c_{t+1}, \qquad \beta > 0.$$

Let τ be the tax rate and write the intertemporal budget constraint of any individual as:

$$p_t c_t + p_{t+1} c_{t+1} = p_t w(1-\tau),$$

where p_t is the price level at time t.

Assume that agents are able to transfer their purchasing power between periods by using money. Thus:

$$M_t = p_t(w(1-\tau) - c_t), \qquad p_{t+1} c_{t+1} = M_t,$$

where M_t is the nominal stock of money issued by the government in any period t. Assume that $M_1 > 0$ and:

$$M_t - M_{t-1} = p_t(g - \tau w),$$

where g is the amount of government purchases of the consumption good at any time t. Finally, denote the inflation rate at time t as:

$$\pi_t = (p_t - p_{t-1})/p_t.$$

I.1. Determine the competitive equilibria of this economy assuming that $g < w$, find the equilibrium consumption levels, real money balances and inflation tax.

I.2. Assuming that g is fixed, what are the optimal levels of inflation and tax rate? How do you interpret?

II. Now assume that government purchases g have a positive effect on individuals' endowment. Think of g as an input for a technology whose output y is a nonrival, non-excludable good and assume that the larger is y, the larger is w. In particular, let:

$$w = w(g) = g^{\alpha}, \qquad \alpha < 1.$$

II.1. Derive the Pareto optimal allocations (in particular, derive the optimal amount g^* of the input g to be used in the defined technology).

II.2. Derive the optimal policy (τ, g) in a competitive equilibrium.

II.3. Now impose the exogenous constraint $\tau \leq \bar{\tau} < \alpha$. What can you say about the optimal policy? You don't need to derive the exact optimal policy: just compare the optimal level of government purchases and inflation in this case with the case in which $\tau \in (0, 1)$. Interpret the results.

III. Discuss the problem of pricing money. In what type of models there is a positive market fundamental for money? How can this be interpreted?