UCLA Department of Economics **Spring 2016**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each question in a separate bluebook.** All three parts will receive equal weight in your grade.

Part 1

Consider a stochastic growth model where a representative household's preferences are given by,

$$E\sum_{t=0}^{\infty}\beta^t\left\{\frac{c_t^{1-\sigma}}{1-\sigma}-A\frac{h_t^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right\},\ \sigma>0, A>0,\ \omega>0,$$

where $0<\beta<1$. For each $t\geq 0$, the technology is given by,

$$c_t+i_t=e^{z_t}\left(u_tk_t\right)^{\theta}h_t^{1-\theta}\ \text{, where}$$

$$k_{t+1}=(1-\delta_t)k_t+i_t$$

$$\delta_t=\delta u_t^{\phi}\ \text{ and}$$

$$z_{t+1}=\rho z_t+\varepsilon_{t+1}\ .$$

As usual, ε is an i.i.d. random variable with mean 0 and variance σ_{ε}^2. The variable u is a choice variable describing the fraction of the capital stock utilized in a given period, where $0\leq u_t\leq 1$. The parameters $\phi>1$ and $0<\delta<1$. Hours worked, h, is also constrained to be in the interval [0,1].

A. Are the preferences of the representative agent consistent with balanced growth? Explain.
B. Carefully state the representative agent's dynamic programming problem for this economy. Obtain a set of equations that determine the optimal stochastic process for $\left\{k_{t+1},c_t,h_t,u_t\right\}_{t=0}^{\infty}$
C. Using the equations obtained in part B, determine if the rate of capital utilization is pro-cyclical or counter-cyclical. Explain [be specific by what you mean by pro (or counter) cyclical].
D. Define a *recursive competitive equilibrium* for this economy. Hint: Consider a market for utilized capital services rather than a capital rental market.

E. The *Frisch elasticity of labor supply* is defined to be the elasticity of hours worked to the wage rate holding the marginal utility of wealth (the Lagrange multiplier for the budget constraint) constant. Derive the Frisch elasticity for your decentralized economy. Make sure you clearly explain your derivation.

F. Derive a log-linear approximation that expresses the percentage deviation of hours worked as a function of the percentage deviation of the wage rate and consumption from their steady states. Assuming consumption is equal to its steady state, use this equation to determine how large is the standard deviation of hours divided by the standard deviation for the wage rate in this economy.

G. Discuss the implications of this model for using the Solow residual or total factor productivity to measure exogenous technical progress.

Part 2.

Time is discrete and the horizon infinite. Every period, a stochastic event s is drawn from a finite set S. The history of events from time zero to time t is denoted $s^t \equiv (s_0, s_1, \ldots, s_t)$. The set of time-$t$ histories is denoted by S^t. The probability of sequence s^t is denoted by $\pi_t(s^t)$. There are two agents $i \in \{1, 2\}$ with CRRA utility function over consumption

$$u_i(c) = \begin{cases} \frac{c^{1-\gamma_i}-1}{1-\gamma_i} & \text{if } \gamma_i \neq 1 \\ \log(c) & \text{if } \gamma_i = 1, \end{cases}$$

for all $c \geq 0$. An agent's inter-temporal utility is, then, $\sum_{t\geq 0} \beta^t \sum_{s^t \in S^t} \pi_t(s^t) u_i\left[c_{it}(s^t)\right]$.

The aggregate endowment in period t after history s^t is a time-invariant function of the current event only, and so is denoted by $y(s_t)$. We assume that, every period, agent of type $i \in \{1, 2\}$ receives a fraction $\bar{n}_i$ of the aggregate endowment. Of course, $\bar{n}_1 + \bar{n}_2 = 1$.

Assume that, at time $t = 0$, after the first event s_0 realizes, a competitive market opens. In this market, agents can buy or sell contingent claims to consumption. We let $q_{0t}(s^t)$ denote the time-zero price of a claim to one unit of consumption at time t after history s^t.

1. Define the problem of agent $i \in \{1, 2\}$ and define an equilibrium. (1pt)

2. Assume that $\gamma_1 = \gamma_2 > 0$. Show that, in equilibrium, agents have constant consumption shares, that is, $c_{it}(s^t)/y_t(s^t)$ remains constant over time. Solve for the consumption share of each agent. (1pt)

3. Now assume that $\gamma_1 = 0$ (risk neutral) and $\gamma_2 = 1$ (log).

    (a) Derive the first-order conditions of agent $i \in \{1, 2\}$. In doing so, keep in mind that the consumption of agent 1 cannot be negative. (1pt)

    (b) Find a formula for the consumption of agent $i = 2$ as a function for the aggregate endowment and the ratio of Lagrange multipliers, λ_2/λ_1, on agents $i \in \{1, 2\}$ respective inter-temporal budget constraints. (1pt)

    (c) How does the consumption and consumption share of the risk-averse agent varies with aggregate output y? Does the risk neutral agent insure the risk averse agent fully against aggregate fluctuations? Explain why. (1pt)

(d) Using the inter-temporal budget constraint of agent 2, derive a one-equation-in-one-unknown problem for the ratio of multipliers, λ_2/λ_1. Argue that a competitive equilibrium exists, and is unique. How does the equilibrium consumption of agent 2 vary with its initial endowment, $\bar{n}_2$. Explain Why. (1pt)

4. For the remainder of the exam, assume that markets open sequentially instead of only once. Every period t and after every history, s^t, agent i receives its endowment $\bar{n}_i y_t(s_t)$, and trade consumption as well as a complete set of one-period ahead Arrow securities. Agents' initial positions in Arrow securities is zero, $a_{i0}(s_0) = 0$. Assume as well that draws of s_t are identically and independently distributed over time.

   (a) Define the agent's problem and define an equilibrium. (1pt)

   (b) Provide a formula for the stochastic discount factor. Sketch a plot of the risk free rate as a function of the aggregate endowment. Sketch a plot of the <u>derivative</u> of the risk free rate as a function of the aggregate endowment. Explain your findings. (1pt)

   (c) Find a formula for the wealth of agent 2 at the beginning of each period, $a_{2t}(s^t)$. (1pt)

   (d) What is the <u>sign</u> of $a_{2t}(s^t)$ when the aggregate endowment, $y(s_t)$, is low? What is the <u>sign</u> of $a_{2t}(s^t)$ when the aggregate endowment, $y(s_t)$, is high? Explain why. (1pt)

Part 3 - Immigration and Wages

Consider an economy with three types of workers: (1) skilled domestic workers, (2) unskilled domestic workers, both of whom reside in the *domestic country*, and (3) unskilled foreign workers, who reside in the *foreign country*, but who can immigrate to the domestic country. There is no physical cost of migrating. Capital markets are perfect.

The skill level of a skilled worker is denoted by h_s. The skill levels of the two types of unskilled workers are normalized to 1. There are S number of skilled workers and U number of unskilled workers in the domestic economy. Denote the number of unskilled workers who immigrate as F. The wage rate for the foreign workers in their home country is given by w, and this is fixed. Each worker is infinitely lived, discounts the future at a constant and identical rate, supplies one unit of labor inelastically, and is risk neutral.

(a) What is the efficiency level of labor input provided by each type of worker? (1 point)

(b) Given the information that you have available, write down the lifetime optimization probem for any worker, denoting the type of worker by i. (1 point)

There is a single physical output good, denoted as y. It is produced by a competitive firm using a constant returns to scale technology which uses the labor inputs described above. .Assume that unskilled domestic and foreign workers are perfect substitutes. Assume that there is a finite and constant substitution elasticity between unskilled domestic and foreign workers and skilled workers.

(c) Write down the production technology for this economy. (2 points)

(d) Derive the marginal products of the inputs. (2 points)

(e) Suppose that the domestic country allowed free immigration. Derive a formula that could be used to determine the number of immigrants. Discuss how the existing number of domestic unskilled and skilled workers affects the number of immigrants. (2 points).

(f) Suppose that the skilled workers controlled immigration. How many unskilled would they allow in? Why? (1 point)

(g) Suppose instead that domestic unskilled workers controlled immigration. How many unskilled workers would they allow in? Why? (1 point)

(h) Suppose that the domestic unskilled workers controlled the number of immigrants, and that the immigrants had to pay a one-time fee to the domestic unskilled in order to immigrate. Derive a formula that could be used to determine the number of immigrants. Is charging a permit fee for immigration Parteo-superior to a policy of no immigration? (6 points)

(i) Virtually every modern trade negotiation provides resources to those workers who are harmed by the trade deal so that the displaced workers can retrain. Describe briefly how the model that you have constructed here to analyze immigration can shed light on job retraining in trade negotiations. (2 points)