UCLA Department of Economics **Fall 2014**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each question in a separate bluebook.** All three parts will receive equal weight in your grade.

Part 1

In this problem you will study **variable capital utilization** in a stochastic growth model. Consider an economy in which a representative household's preferences are given by,

$$E\sum_{t=0}^{\infty}\beta^t \log c_t \ ,$$

where $0 < \beta < 1$. For each $t \geq 0$, the technology is given by,

$$c_t + i_t = e^{z_t}\left(u_t k_t\right)^{\theta} h_t^{1-\theta} \ \text{, where}$$

$$k_{t+1} = (1-\delta)k_t + i_t \ \text{ and}$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \ .$$

As usual, ε is an i.i.d. random variable with mean 0 and variance σ_ε^2. The variable u is a choice variable describing the fraction of the capital stock utilized in a given period, where $0 \leq u_t \leq 1$. Hours worked, h, is also constrained to be in the interval [0,1].

A. Carefully state the representative agent's dynamic programming problem for this economy. Obtain expressions for the optimal values of u and h as functions of the state variables. Does the rate of capital utilization vary depending on the technology shock, z. If so, is utilization pro-cyclical or counter-cyclical? Explain.
B. Suppose now that the rate of depreciation is not a constant, but depends on the rate of capital utilization. In particular, suppose that $\delta_t = \delta u_t^{\phi}$, where $\phi > 1$ and $0 < \delta < 1$. Repeat part (A) for this new economy.
C. Derive a log-linear approximation to the Euler equation for the economy of part B. Express this (at least implicitly) as a function of $z_t, Ez_{t+1}, \tilde{k}_t, \tilde{k}_{t+1}$ and $E\tilde{k}_{t+2}$, where $\tilde{k}_t \equiv \log k_t - \log \bar{k}$. A bar above a variable denotes a nonstochastic steady state value. Describe how one can solve this to obtain an expression $\tilde{k}_{t+1} = a\, z_t + b\,\tilde{k}_t$. What is the transversality condition and how do you guarantee that it is satisfied?

D. Define a *recursive competitive equilibrium* for the economy in part (B). Hint: Consider a market for utilized capital services rather than a capital rental market.

E. Discuss the implications of the model in part (B) for using the Solow residual or total factor productivity to measure exogenous technical progress.

Part 2

1 International Income and Productivity Comparisons

Consider a two country, three good economy in which time is discrete and denoted $t = 0, 1, 2, 3, \ldots$. The first good, the aggregate output of which we denote y_{Tt}, is freely traded across countries. The consumption of this good in the first country is denoted c_{Tt} and in the second country by c^*_{Tt}. The second good is a non-traded good produced and consumed only in country one, with aggregate output y_{Nt} and consumption c_{Nt}. The third good is a non-traded good produced and consumed only in country two, with aggregate output y^*_{Nt} and consumption c^*_{Nt}.

Let the utility of the representative consumer in country 1 be given by

$$\sum_{t=0}^{\infty} \beta^t \left[\theta \log(c_{Tt}) + (1-\theta)\log(c_{Nt})\right]$$

and in country 2 by

$$\sum_{t=0}^{\infty} \beta^t \left[\theta \log(c^*_{Tt}) + (1-\theta)\log(c^*_{Nt})\right]$$

In each country, the representative agent is endowed with one unit of labor each period which can be allocated to the production of the traded good or the local non-traded good. The productivity of labor in producing the traded good in country one is given by A_{Tt} and in country two by A^*_{Tt}. The productivity of labor in producing the non-traded good in each country is fixed at one every period. If we let $l_{Tt} \geq 0$ and $l_{Nt} \geq 0$ with $l_{Tt} + l_{Nt} = 1$ denote the allocation of labor in country 1 and $l^*_{Tt} \geq 0$ and $l^*_{Nt} \geq 0$ with $l^*_{Tt} + l^*_{Nt} = 1$ denote the allocation of labor in country 2, then aggregate output of the traded good is given by $y_{Tt} = A_{Tt} l_{Tt} + A^*_{Tt} l^*_{Tt}$. Aggregate output of the non-traded good in each country is given by $y_{Nt} = l_{Nt}$ in country one and $y^*_{Nt} = l^*_{Nt}$ in country two.

The resource constraints in this economy are $c_{Tt} + c^*_{Tt} = y_{Tt}$ for the traded good and $c_{Nt} = y_{Nt}$ for the non-traded good in country 1 and $c^*_{Nt} = y^*_{Nt}$ in country two.

Part A: Define a feasible allocation, define a price system including prices for the three goods and wages in each country at every date, and define a competitive equilibrium.

Part B: Pose a social planning problem that can be used to characterize the competitive equilibrium allocations in this economy as a function of the Pareto weights on the representative agent in country 1 and country 2.

Part C: Use either your definition of competitive equilibrium from Part A or the social planning problem from Part B to characterize the relative wages in country one and country two, w_t/w_t^* as a function of the relative productivity of labor in producing the traceable good in each country, A_{Tt}/A_{Tt}^* in equilibrium for all dates such that l_{Tt} and l_{Tt}^* are both greater than or equal to zero.

Part D: Use either your definition of competitive equilibrium from Part A or the social planning problem from Part B to characterize the relative price of the non-traded good in country one and country two, p_{Nt}/p_{Nt}^* as a function of the relative productivity of labor in producing the traceable good in each country, A_{Tt}/A_{Tt}^* in equilibrium for all dates such that l_{Tt} and l_{Tt}^* are both greater than or equal to zero.

Part E: We compute the price level in each country at each date t as a geometric weighted average of the prices of the traded and non-traded goods. In country one, this is $P_t = p_{Tt}^{\alpha} p_{Nt}^{1-\alpha}$ and, in country two $P_t^* = p_{Tt}^{*\alpha^*} p_{Nt}^{*(1-\alpha^*)}$, where we set $p_{Tt} = p_{Tt}^*$ for all t since this good is freely traded. Let α and α^* be the share of total consumption expenditure in country 1 and 2 respectively spent on the traded good. (Note from the specification of the utility functions above that we can compute this share as a function of parameters). What is the ratio of price indices P_t/P_t^* as a function of the relative productivity of labor in producing the traceable good in each country, A_{Tt}/A_{Tt}^* in equilibrium for all dates such that l_{Tt} and l_{Tt}^* are both greater than or equal to zero?

Part F: Use your answers to Part C and Part E to construct the following plot. Imagine that at different dates, the ratio of relative productivities of labor in producing the traceable good in each country, A_{Tt}/A_{Tt}^*, varies considerably. Correspondingly, the ratio of relative wages w_t/w_t^* and relative price levels P_t/P_t^* will also differ. If we were to produce a scatter plot using data from different dates with the log of relative wages on the x-axis and the log of relative price levels on the y-axis, what would this plot look like?

Part 3

This question is worth **40** points total.

Consider the following monetary model

(1.1) $$i_t - E_t[\pi_{t+1}] - \rho(E_t[y_{t+1}] - y_t) = r,$$

(1.2) $$m_t - p_t + i_t = y_t,$$

(1.3) $$y_t = \bar{y} + e_t.$$

In this model i_t is the money interest rate, p_t is the log of the price level, $\pi_t = p_t - p_{t-1}$ is the log difference of the price level between periods t and $t-1$, y_t is the log of real GDP, $\bar{y}$ is the log of potential output, m_t is the log of the quantity of money, measured in dollars, e_t is a fundamental shock to aggregate supply and ρ and r are parameters. Assume further that

(1.4) $$E_t(e_s) = 0, \quad s > t.$$

A. (4 points) Equation (1.1) is often derived by linearizing the Euler equation of a representative agent. What is the interpretation of the parameters ρ and r? If the representative agent had logarithmic preferences, what would that imply for the value of ρ?
B. (4 points) If you were to estimate this equation and find that r was negative, would that be a problem for your interpretation of the equation as an Euler equation? Explain your answer.
C. (4 points) Equation (1.2) represents a demand-for-money function. Explain what is meant by zero degree homogeneity of the money demand function. Use this equation to illustrate your answer.
D. (4 points) Assume that the Central Bank follows the interest rate rule,

(1.5) $$i_t = \bar{i}, \quad \text{for all } t$$

Find an expression for the expected inflation rate in a rational expectations equilibrium as a function of $\bar{i}, \rho, r$ and e_t. Is the rational expectations equilibrium unique? If not, explain why not.

E. (4 points) Assume now that the Central Bank follows the following money supply process,

(1.6) $$m_t = m_{t-1} + \mu.$$

Find an expression for the expected inflation rate in a rational expectations equilibrium as a function of μ, ρ and e_t. Is this equilibrium unique? Explain your answer paying particular attention to any differences with your answer to Part D.

F. (4 points) Explain what is meant by the Taylor Principle. Does this help to explain your answer to part D?

G. (6 points) Let

(1.7) $$X_t = \left[E_t\left(\pi_{t+1}\right), E_t\left(y_{t+1}\right), i_t, y_t, \pi_t, p_t\right]^T$$

be a vector of variables, let

(1.8) $$\eta_t = \left[\eta_t^1, \eta_t^2\right]^T$$

be a vector of non-fundamental shocks, let c be a 6 x 1 vector of constants and assume that the money supply is constant and equal to $\bar{m}$. Show how to write this model in the form

(1.9) $$AX_t = BX_{t-1} + \psi u_t + \Pi \eta_t + c.$$

What are the elements of the matrices *A*, *B*, ψ, Π and c?

H. (6 points) Explain how you would use the QZ decomposition to find a solution to this model.

I. (4 points) Explain what is meant by indeterminacy of a rational expectations equilibrium. Can the representative agent growth model, in the absence of money, ever display indeterminacy? Explain your answer by drawing on your knowledge of general equilibrium theory.