UCLA Department of Economics **Fall 2012**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part.

Answer each part in a separate bluebook. All three parts will receive equal weight in your grade.

Part I

1. Consider the following problem solved by a social planner:

$$\max_{\{n_t, h_t, k_{t+1}\}_{t=0}^{\infty}} E\sum_{t=0}^{\infty} \beta^t [\log c_t + n_t A \log(1-h_t)]$$

subject to

$$c_t + i_t + m_t \le e^{z_t} h_t k_t^{\theta} n_t^{1-\theta}$$

$$0 \le n_t \le 1$$

$$m_t = \frac{\alpha}{2}(n_t - n_{t-1})^2 \ , \ \alpha \ge 0$$

$$k_{t+1} = (1-\delta)k_t + i_t$$

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \ , \ \varepsilon_{t+1} \sim N(0, \sigma^2)$$

n_{-1} and k_0 given.

The variables c_t, i_t, m_t, h_t, n_t, k_t, z_t are time t values of consumption, investment, costs associated with moving between market and non-market sector, length of a work shift, employment rate, capital stock and the technology shock, respectively. The interpretation is that the planner chooses both employment (fraction of people who work), n, and hours worked per person, h. If $\alpha > 0$, it is costly to hire or fire workers.

A. Write down the Bellman's equation associated with this planner's problem.

B. Derive set of equations that characterize a sequence $\{c_t, h_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ that solves this problem. Be sure that you have the same number of equations as unknowns.

C. Describe a strategy for calibrating this economy (assigning values to $\beta, A, \theta, \alpha, \delta, \rho$, and σ). What statistics do you need from actual data in order carry out your strategy?

D. Prove that if $\alpha = 0$, h_t is equal to a constant (that it is not affected by the state of the economy).

E. Define a *recursive competitive equilibrium* for the case when $\alpha = 0$.

F. Prove that the equilibrium allocations are the same as those chosen by the social planner in this case ($\alpha = 0$).

1 Question 2

1.1 Two Sector Growth

The maximization problem for a representative household is:

$$\max E \sum_{t=0}^{\infty} \beta^t \{\alpha \ln(C_{1t}) + (1-\alpha)\ln(C_{2t})\}$$

The resource constraints are:

$$C_{1t} + I_{1t} = A_1 K_{1t}^{\theta} (X_{1t} L_{1t})^{1-\theta}$$

$$C_{2t} + I_{2t} = A_2 K_{2t}^{\theta} (X_{2t} L_{2t})^{1-\theta}$$

The accumulation equations are:

$$K_{1t+1} = I_{1t} + (1-\delta) K_{1t}$$
$$K_{2t+1} = I_{2t} + (1-\delta) K_{2t}$$

Technological change is given by:

$$X_{1t} = (1+\gamma_1)^t, X_{2t} = (1+\gamma_2)^t, \gamma_1, \gamma_2 > 0$$

The constraint for labor is:

$$L_1 + L_2 \leq 1$$

(A) Define a stationary recursive equilibrium for this economy, and let lower case letters be stationary variables. Let the numeraire be c_1. Denote the price of c_2 as p. Explain why c_1 and i_1 sell for the same price, and why c_2 and i_2 sell for the same price. Present a formula for the stationary relative price, and also the price for the economy when you add growth back in to the model.

(B) Suppose that the economy is in a steady state, and α declines. What happens to the quantities of steady state K_1 and K_2, L_1 and L_2, and steady state wage and rental prices of capital? What happens to the relative price of the output of good 2? What happens to GDP, measured as $y_1 + py_2$? Explain your answers. (Here, you should state whether the variable of interest increases, declines, or remains constant, and the economic forces generating the result).

(C) Suppose that the government taxes consumption of good 2, and rebates the tax revenue back to the household. What do you think would happen to steady state values of total output, price of good 2, wage, rental rates, and steady state values of capitals and labor inputs? Suppose that the two goods were perfect substitutes in utility. What would happen to allocation of production inputs across the two sectors? Why is constant returns to scale important for your answer?

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Question 3

Fiscal Policy in an Overlapping Generations Model:

Consider the following overlapping generations model. Time is discrete and denoted $t = 1, 2, 3, \ldots$. In period 1, there is an initial cohort of old agents of measure 1. These agents are endowed with the initial capital stock k_0 and the initial stock of government debt b_0. These agents have utility which is strictly increasing in consumption c_1^o. Every period $t \geq 1$, a cohort of measure 1 of two-period lived agents is born. These agents are each endowed with 1 unit of labor when young, which they supply inelastically. These agents have utility over their consumption while young, c_t^y, and their consumption while old c_{t+1}^o, given by

$$\log(c_t^y) + \log(c_{t+1}^o)$$

Each period, there are firms that rent physical capital and hire labor to produce output according to

$$y_t = Ak_{t-1} + l_t$$

The firm pays rental payments $r_t k_{t-1}$ to the owners of the capital stock at t (the old at t) and wage payments $w_t l_t$ to the young at t. The firm chooses k_{t-1} and l_t to maximize profits given rental rate r_t and wage rate w_t each period.

The government consumes spending every period g_t. The government finances this expenditure with taxes on labor and with debt. The government collects $\tau_t w_t l_t$ in tax revenue from young workers' wages and these workers take home $(1-\tau_t)w_t l_t$ in after tax wages. The government sells debt b_t at price q_t. In period t, the debt pays off b_t in goods in period $t+1$. The government budget constraint is given by

$$q_t b_t + \tau_t w_t l_t = g_t + b_{t-1}$$

for al $t \geq 1$.

Assume that there is complete depreciation of physical capital each period. The young choose consumption, bond holdings, and holdings of physical capital. The old consume their income from physical capital and bonds.

An allocation in this environment is a sequence of consumption for young and old, together with a capital stock and labor supply $\{c_t^y, c_t^o, k_t, l_t\}_{t=1}^{\infty}$. A feasible allocation satisfies

$$c_t^y + c_t^o + k_t = Ak_{t-1} + l_t$$

and $l_t = 1$, $k_t, c_t^y, c_t^o \geq 0$ for all $t \geq 1$.

Part A: Define a competitive equilibrium (without money) in this environment. If the young at t hold strictly positive amounts of bonds and capital, what is the price of bonds q_t?

Part B: Assume that $A > 1$. Construct a numerical example with constant government expenditure g_t and a constant tax rate τ_t such that government debt b_t stays constant at its initial value b_0. Show that k_t is constant in this equilibrium for $t \geq 1$. Show that the consumption of the young is constant for $t \geq 1$ and that the consumption of the old is constant for $t \geq 2$. Is output constant for $t \geq 2$? Is it necessarily the case that τ_t

Part C: Show that the constant value of capital, output, and consumption of the old all decline if b_0 is higher in your example. Show that there is a maximum value of b_0 that is consistent with an equilibrium in which g_t, τ_t, and $b_t = b_0$ are all constant.

Part D: Now assume that $A < 1$. Is it possible to construct an example equilibrium in which g_t, τ_t and $b_t = b_0 \geq 0$ are all constant and government runs a deficit every period $\tau_t < g_t$? If you say yes, construct such an equilibrium. If you say no, say why not.

Qualifying Exam in Macroeconomic Theory (Addendum)

Question 3

Part B:

[Is it necessarily the case that] tax revenue is greater than government spending in this equilibrium?

(Hint: opposite scenario to Part D)