September 24, 2008

UCLA

Department of Economics

Ph. D. Preliminary Exam

Macroeconomic Theory
(Fall 2008)

Instructions:

- You have 4 hours for the exam.
- Answer all 3 questions. All questions are weighted equally.
- Use a SEPARATE blue book to answer each question.
- Calculators and other electronic devices are not allowed.

Macro Comprehensive Exam
Fall 2008

Question 1

1. Consider a two sector neoclassical growth model with manufactured goods and services. Manufactured goods are produced using the production function, $m_t = z_{1t}\gamma^{(1-\theta)t}k_t^{\theta}h_{1t}^{1-\theta}$ where $\gamma > 1$ and $0 < \theta < 1$. Services are produced only from labor, $s_t = Az_{2t}\lambda^t h_{2t}$, where $A > 0$ and $\lambda > 1$. The parameters γ and λ are not necessarily equal to each other. The shocks to technology, z_{1t} and z_{2t}, are governed by the stochastic process, $\log z_{i,t+1} = \rho_i \log z_{i,t} + \varepsilon_{i,t+1}$ $(i = 1,2)$ where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$. Manufactured goods can be consumed or invested, $c_{1t} + x_t \le m_t$, where x_t is investment and $k_{t+1} = (1-\delta)k_t + x_t$. Services can only be consumed, $c_{2t} \le s_t$. The preferences of the representative agent are given by $E\sum_{t=0}^{\infty}\beta^t\left(\alpha \log c_{1t} + (1-\alpha)\log c_{2t}\right)$. Labor assigned to the two sectors must satisfy $h_{1t} + h_{2t} \le 1$. The initial stock of capital, k_0, is given.

A. Characterize the steady state growth paths for c_{1t}, c_{2t}, k_t, m_t, h_{1t}, and h_{2t}.

B. Describe how you might calibrate this economy to U.S. data using the calibration methods that have been employed in the real business cycle literature. In particular, you need to assign parameters to γ, λ, θ, A, δ, α, ρ_1, ρ_2, σ_1, and σ_2. Be specific about what information you would need and how that information would lead to particular parameter values.

C. Define a recursive competitive equilibrium for this economy (let $\gamma = \lambda = 1$).

D. Derive the Euler equation that determines the equilibrium path for the stock of capital for the economy of part C. Log-linearize this equation and explain how an equilibrium law of motion for capital can be obtained from this. Is the transversality condition satisfied? Explain.

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An Economy with Infrastructure Capital

Question 2

(a) Consider an economy with a representative consumer whose preferences are defined by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma},$$

where C_t is consumption and the parameters satisfy $0 < \beta < 1$ and $\sigma > 0$. The consumer provides one unit of labor inelastically.

There is a competitive industry which operates the production technology:

$$y_t = AK_t^{\alpha} N_t^{1-\alpha} G_t^{1-\alpha},$$

where K_t is capital, N_t is aggregate labor supply, G_t is infrastructure capital, the parameter α satisfies $0 < \alpha < 1$, and $A > 0$ is constant. Both types of capital depreciate completely every period. The resource constraint of the economy is given by:

$$C_t + K_{t+1} + G_{t+1} = AK_t^{\alpha} N_t^{1-\alpha} G_t^{1-\alpha}.$$

- Provide a Bellman equation for the social planning problem. The number of state variables should be as low as possible.
- Which ratio of production capital and infrastructure capital K_{t+1}/G_{t+1} will be chosen by the planner?
- What is the growth rate of output and consumption chosen by the planner?
- Characterize the transitional dynamics of the planning solution.

(b) In this part of the question, we will compare the planning outcome to the outcome in a competitive economy in which infrastructure is financed through taxes. Firms are renting physical capital and labor at rates r_t and w_t from households, respectively, while infrastructure is provided at no cost for firms. The government levies a constant proportional income tax τ on households to finance infrastructure investment. The household's budget constraint is:

$$C_t + K_{t+1} = (1-\tau)(r_t K_t + w_t),$$

and infrastructure investment is given by:

$$G_{t+1} = \tau(r_t K_t + w_t N_t).$$

- Define an equilibrium for this economy.
- What is the balanced growth rate of this economy?
- Can the government choose a τ that reproduces the social planning outcome? If not, is there an alternative arrangement for financing infrastructure that could achieve the optimal outcome?

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Consider an infinite horizon endowment economy in which agents face idiosyncratic uncertainty regarding their preferences over the timing of their consumption. Number the periods $t = 0, 1, 2, \ldots$. Each period, for each consumer, an event θ_t is realized with $\theta_t \in \Theta = \left\{\theta^1, \ldots, \theta^N\right\}$. These events are i.i.d. across time and across consumers with probabilities $\rho(\theta^n)$. We assume that $\rho(\theta^n)$ also represents the fraction of agents who receive shock θ^n. Let $h^t = (\theta_0, \theta_1, \ldots, \theta_t)$ denote the history of events an agent has experienced through date t and let $\pi_t(h^t)$ denote the date zero probability of this history. Let $\theta_t(h^t)$ denote the last element of history h^t.

An allocation in this economy is a sequence of consumption plans

$$\left\{c_t(h^t)\right\}_{t=0}^{\infty}.$$

The resource constraints in this economy are that

$$\sum_{h^t \in \Theta^t} c_t(h^t)\pi_t(h^t) = Y_t$$

for all t, with $\{Y_t\}_{t=0}^{\infty}$ a deterministic sequence of aggregate endowments.

Consumers have preferences given by

$$\sum_{t=0}^{\infty} \beta^t \sum_{h^t \in \Theta^t} \theta_t(h^t) \log(c_t(h^t))\pi_t(h^t)$$

a) Full Information Social Optimum part 1

Write down the first order conditions characterizing the allocation that maximizes the utilitarian social welfare function

$$\sum_{t=0}^{\infty} \beta^t \sum_{h^t \in \Theta^t} \theta_t(h^t) \log(c_t(h^t))\pi_t(h^t)$$

subject to the resource constraints. Does your solution for social welfare here depend on the distribution of shocks θ if we hold the mean of the shocks θ constant at one?

b) Bond economy

Consider now an incomplete markets economy in which, at each date t, agents only trade shares of the aggregate endowment Y. Let q_t denote the price of this share and $s_t(h^t)$ the quantity purchased by an agent with history h^t. Given this notation, agents have budget constraints

$$c_t(h^t) + q_t\left(s_t(h^t) - s_{t-1}(h^{t-1})\right) = s_{t-1}(h^{t-1})Y_t$$

for all $t > 0$ and h^t. At date 0, assume that agents all start with initial shareholdings $s_{-1} = 1$ so they face constraint

$$c_0(h^0) + q_0(s_0(h^0) - 1) = Y_0$$

Agents are constrained to have non-negative shareholdings $s_t(h^t) \geq 0$.

Assume that Y_t is constant for all t.

Equilibrium consumption has the form

$$c_t(h^t) = Bs_{t-1}(h^{t-1})(q_t + Y)$$

Show how to solve for the parameter B.

Next, show that the there is an equilibrium with a constant share price $q_t = q$ every period. Describe how to solve for this constant share price.

d) Bond Economy Part 2

Prove that the constant share price satisfies

$$\frac{1}{\beta} > \frac{q+Y}{q}$$