UCLA

Department of Economics

Ph. D. Preliminary Exam

Macroeconomic Theory
(FALL 2007)

Instructions:

- You have 4 hours for the exam.
- Answer all 3 questions. All questions are weighted equally.
- Use a SEPARATE blue book to answer each question.
- Calculators and other electronic devices are not allowed.

Question 1

Consider the following two-sector neoclassical growth model studied by Boldrin, Christiano and Fisher, *AER* (2001).

A representative agent has preferences defined over sequences of consumption, c_t, and hours worked, h_t. In particular, preferences are given by $E\sum_{t=0}^{\infty}\beta^t\left[\log(c_t - bc_{t-1}) - h_t\right]$, where $0 < \beta < 1$ and $b \geq 0$.

There are two technology sectors. One produces consumption goods (subscript *c*) and the other investment goods (subscript *i*):

$$c_t \leq k_{c,t}^{\alpha}(z_t h_{c,t})^{1-\alpha}$$

$$k_{c,t+1} + k_{i,t+1} \leq k_{i,t}^{\alpha}(z_t h_{i,t})^{1-\alpha} + (1-\delta)(k_{c,t} + k_{i,t})$$

The technology shock, z_t, is assumed to evolve according to $\log z_{t+1} = \gamma \log z_t + \varepsilon_{t+1}$ where ε is i.i.d. with mean zero and variance σ^2.

The hours allocation (h_t, $h_{c,t}$, $h_{i,t}$) is made ***before*** agents observe z_t. In addition, once capital is installed in a particular sector, it cannot be moved.

A. The preferences assumed here are linear in hours worked. Explain how this follows from preferences that are separable in leisure if labor is assumed to be indivisible and there is a market for employment lotteries.

B. State the dynamic programming problem solved a social planner for this economy. Be complete and precise. *Note: This and all subsequent questions refer to the original economy described above, not an economy with employment lotteries.*

C. Derive a set of equations, including first order necessary conditions, that characterizes a solution to the problem of part B.

D. Is the solution characterized in part C consistent with *balance growth*? Thoroughly justify your answer and be sure to explain what is meant by *balanced growth.*

E. Derive a log-linear approximation of the two resource constraints. Explain your work.

F. Describe a decentralization of this economy and define a *recursive competitive equilibrium.*

Question 2. A Monetary Economy

Consider a cash-credit goods economy, in which lower case objects are household choices, and upper-case objects are per-capita aggregate variables. Preferences are given:

$$\max \sum_{t=0}^{\infty} \beta^t \{\alpha \log(c_{1t}) + \gamma \log(c_{2t}) + (1 - \alpha - \gamma) \log(1 - n_t)\},$$

Aggregate production is given by:

$$C_{1t} + C_{2t} = Y_t = AN_t, A \geq 1$$

The CIA constraint for the representative household is given by:

$$m_t \geq p_t c_{1t}$$

The wealth constraint for the household is given by:

$$m_t + b_t \leq m_{t-1} - p_{t-1}c_{1t-1} - p_{t-1}c_{2t-1} + W_{t-1}n_{t-1} + R_{t-1}b_{t-1} + T_t$$

where B is one period government bond holdings, R is the gross interest rate, W is the wage rate, and T is a lump sum tax-transfer. The government's monetary policy is to adjust money balances to keep the interest rate constant at $\bar{R}$. Assume that the initial stock of bonds is zero.

(1) Define a competitive equilibrium, and write down the equations characterizing a competitive equilibrium for this economy. Why is it that the gross nominal interest rate is bounded below at $R_t = 1$? Does this mean that the real interest rate cannot be negative in this economy?

(2) What happens to n as R rises? Prove your result.

(3) What value of R maximizes consumer welfare? Denote this interest rate as R^*. Does there exist a government policy such that the competitive equilibrium coincides with the social planner's problem at R^*? If so, prove this result.

(4) Suppose that the government fixes the interest rate at R_o in odd periods and R_e in even periods. Calculate the equilibrium allocations in the two types of periods. What can you say about money growth from odd to even periods, and from even to odd periods?

(5) Suppose that the efficiency parameter, A, depended on government spending:

$$A = A(g), \frac{dA}{dG} > 0$$

Suppose also that government expenditures could only be financed with seignorage (revenue from money creation). Would the government policy that maximizes welfare in (3) still be the optimal policy under this assumption about A? If not, why not? Make any assumptions you feel are necessary for your analysis.

Macro Comprehensive Exam Question 3
Fall 2007

In this question, we will consider an infinite horizon economy in which output follows an exogenously given stochastic process. In part a), the logarithm of the level of output follows an AR1 process with a deterministic time trend. In part f), the logarithm of the growth rate of output is *i.i.d.* We assume that output is not storable, so that the consumption of the representative agent is equal to output. Hence, in equilibrium $c_t = y_t$ in every period and in every state of nature. The preferences of the representative consumer are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\alpha} c_t^{1-\alpha}.$$

We ask about the yields on two securities: a long maturity riskless claim and a long-maturity risky claim. Let $\bar{q}_t$ denote the price, in terms of consumption at date 0 of a riskless claim to one unit of consumption at date t. Define the per-period yield on this security as

$$\bar{r}_t = \frac{1}{t} \log\left(\frac{1}{\bar{q}_t}\right) = -\frac{1}{t} \log(\bar{q}_t).$$

Let $\tilde{q}_t$ denote the price, in terms of consumption at date 0 of a claim to y_t units of consumption at date t. Define the expected per-period yield on this security as

$$\tilde{r}_t = E_0 \frac{1}{t} \log\left(\frac{y_t}{\tilde{q}_t}\right) = E_0 \frac{1}{t} \left[\log y_t - \log \tilde{q}_t\right].$$

We price these claims in this economy assuming that there is a large set of state-contingent securities. In particular, we assume that for each date t and each possible realization of output y_t, there is a security traded at date 0 that pays off one unit of consumption if and only if y_t is realized at date t. Letting $f_t(y_t)$ be the probability density of possible realizations of output at t, we write the representative consumer's utility maximization problem as follows. The consumer chooses consumption $c_t(y_t)$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{1-\alpha} \int_{y_t} c_t(y_t)^{1-\alpha} f_t(y_t) dy_t$$

subject to the date 0 budget constraint

$$\sum_{t=0}^{\infty} \int_{y_t} q_t(y_t) c_t(y_t) f_t(y_t) dy_t \leq \sum_{t=0}^{\infty} \int_{y_t} q_t(y_t) y_t f_t(y_t) dy_t.$$

Market clearing in equilibrium requires that $c_t(y_t) = y_t$ for all t and y_t.

There is one point per part except for the last one (part i) which is worth two points.

Part a) Assume that

$$\log y_{t+1} = t \log g + \log z_{t+1}$$

where

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}.$$

Assume that $\log z_0$ is given, that $0 < \rho < 1$ and that ε_{t+1} is normally distributed with mean 0 and standard deviation σ. What is

$$\lim_{t \to \infty} \frac{1}{t} E_0 \log y_t?$$

Part b) What is

$$\lim_{t \to \infty} \left(E_0 y_t\right)^{1/t}?$$

Recall that if x is lognormally distributed, so that $\log x$ is normal with mean m and variance s^2, then $Ex = \exp\left(m + s^2/2\right)$.

Part c) Use the representative consumer's problem and market clearing to give a formula for $\log \bar{q}_t$ in terms of $\log y_0$, the first and second moments of $\log y_t$ (you can denote these by m_t and s_t^2) and the parameters β and α of the representative consumer's utility function.

Part d) Do the same for $\tilde{q}_t$.

Part e) Compute the per-period yields on these two assets as their maturity gets long, that is, compute

$$\lim_{t \to \infty} \frac{1}{t} \bar{r}_t \text{ and } \lim_{t \to \infty} \frac{1}{t} \tilde{r}_t.$$

What is the risk premium (in terms of per-period yields) on the risky asset relative to the risk-free asset as the maturity gets long?

Part f) Now consider an alternative stochastic process for output in this economy. Let z_0 be given and let

$$\log y_{t+1} = t \log g + \log z_{t+1}$$

where

$$\log z_{t+1} = \log z_t + \varepsilon_{t+1}.$$

Assume that ε_{t+1} is normally distributed with mean 0 and standard deviation σ.What is

$$\lim_{t \to \infty} \frac{1}{t} E_0 \log y_t \text{ and } \lim_{t \to \infty} \left(E_0 y_t\right)^{1/t}?$$

Part g) Use the representative consumer's problem and market clearing to give formulas for $\log \bar{q}_t$ and $\log \tilde{q}_t$ in terms of $\log y_0$, the first and second moments of $\log y_t$ (you can denote these by m_t and s_t^2) and the parameters β and α of the representative consumer's utility function.

Part h) Compute the per-period yields on the two assets as their maturity gets long, that is, compute

$$\lim_{t\to\infty} \frac{1}{t}\bar{r}_t \text{ and } \lim_{t\to\infty} \frac{1}{t}\tilde{r}_t.$$

What is the risk premium (in terms of per-period yields) on the risky asset relative to the risk-free asset as the maturity gets long?

Part i) Explain why the risk premium on the risky asset is larger in one economy versus another. Relate your results to the implications of the fact that we do see in data that long-maturity risky assets pay a significant risk premium (in terms of annual yields) for what the variance of the logarithm of consumption around trend in the long-run must look like.