Macroeconomic Theory
Ph.D. Qualifying Examination
UCLA Department of Economics

September 2006

Instructions: You have 4 hours to complete the exam. Answer all six questions. Each question has equal weight. **Answer each question in a separate blue book.**

Warning: You may be under time pressure for this exam. Budget your time accordingly and do not waste too much time on any one question. The average time for each question is 40 minutes.

Question 1

1. This problem concerns an economy with a representative family that lives forever. The family can produce consumption goods or investment goods. The production possibilities frontier is described by a function:

$$C_t = G\left(K_t, A_t, L_t, I_t\right)$$

where K_t is the stock of capital L_t is employment, C_t is consumption, I_t is investment and A_t is given by the equation

$$A_t = (1+g)^t .$$

The family maximizes utility:

$$U_t = \sum_{t=1}^{\infty}\left(\frac{1}{1+\rho}\right)^{t-1}\left(b\log\left(C_t\right) + (1-b)\log\left(1-L_t\right)\right)$$

subject to the sequence of constraints

$$K_0 = \bar{K}, \qquad K_{t+1} = K_t\left(1-\delta\right) + I_t, \qquad t = 1, 2...$$

1. Define what is meant by a "balanced growth path". What conditions do you need to impose on the function $G()$ to guarantee the existence of a balanced growth path. **(2 points)**

2. Suppose that $G()$ is given by the function:

$$G\left(K, L, A, I\right) = \left\{\left[\left(aK^{\theta} + (1-a)\left(AL\right)^{\theta}\right)^{\frac{1}{\theta}}\right]^{\frac{1}{1+\eta}} - I^{\frac{1}{1+\eta}}\right\}^{1+\eta}$$

   Write a pair of first order conditions that characterize the solution to the model; one for the intra temporal choice of labor and the other characterizing the intertemporal choice of capital. **(4 points)**

3. Let k, c, i and y be defined as:

$$y = \frac{Y}{(1+g)^t}, \quad i = \frac{I}{(1+g)^t}, \quad k = \frac{K}{(1+g)^t} \quad, c = \frac{C}{(1+g)^t}.$$

   Write down four equations using the variables k, c, y and L that characterize the equilibria of the transformed model. *HINT*: these are the production possibilities frontier, the two first order conditions and the capital accumulation equation. **(2 points)**

4. What is meant by a determinate equilibrium for this model? Give an explanation of the relevance of determinacy to the way that you would compute a solution to the equations of the model. What restrictions would you need to place on the parameters of the model in order to guarantee that the equilibrium of this representative agent economy is determinate? **(2 points)**

Question 2

Consider the two period overlapping generations model (with no population growth) in which agents live for two periods. Agents have preferences given by:

$$U = -n_t^2 + c_{t+1}$$

where n_t is labor supplied to the market when young and c_{t+1} is consumption when old. (In this model the young work and the old consume). Output is produced from labor with the technology:

$$y_t = n_t$$

1. **(3 points)** Define dynamic efficiency in the context of an infinite horizon model of this kind.

2. **(4 points)** Prove that in the absence of money the unique competitive equilibrium of the model is dynamically inefficient.

3. **(3 points)** Explain how the introduction of money can potentially restore dynamic efficiency.

Question 3

The maximization problem for a representative household is:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \{\alpha_t \ln(c_{1t}) + \phi(1 - l_t)\}$$

The resource constraint for the consumption good is:

$$C_t \leq A_1 K_{1t}^{\theta} (X_{1t} L_{1t})^{\nu}, 0 < \nu < 1 - \theta$$

The resource constraint for the investment good is:

$$I_t \leq A_2 K_{2t}^{\theta} (X_{2t} L_{2t})^{1-\theta}$$

The law of motion for the capital stock is given by:

$$K_{t+1} = I_t + (1 - \delta) K_t$$

The remaining constraints are given by:

$$K_t = K_{1t} + K_{2t}$$

$$X_{1t} = (1 + \gamma_1)^t, X_{2t} = (1 + \gamma_2)^t, \gamma_1 > 0, \gamma_2 > 0$$

The random variable α has strictly positive support.

1. Define a stationary recursive equilibrium for this economy, and let lower case letters be stationary variables. Let the numeraire be new investment, I. Denote the price of C as p_1, denote the price of capital, K, as p_2. Explain why I and K sell for the same price. Present a formula for the relative price of consumption in the stationary economy and in the non-stationary economy. Explain why the size of the parameter ν helps determine the relative price of consumption. Explain (in words) why the competitive equilibrium allocations and the quantities chosen by a benevolent social planner coincide. How does variation in α impact the first order condition for the allocation of time? Discuss the economic intuition of the impact of changes in α in this FOC **(6 points)**

2. Suppose that the economy is in a steady state with $\alpha = 1$ and ν jumps to the value $1 - \theta$. What should happen to the values of stationary steady state K, C, p? What

happens to GDP, measured as $Y_1 + pY_2$. Explain your answers. Sketch what you think the time paths of C, I, p, and L will be on the transition to the new steasy state. (Note there is no closed form solution, but you should be able to graph time paths for the variables and explain the economic forces generating these paths between the old and new steady states). **(4 points)**

Question 4

Consider an economy with mass one of identical consumers whose preferences are defined by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \log(C_t),$$

where c_t is consumption, and the parameter β satisfies $0 < \beta < 1$. Each consumer inelastically supplies one unit of labor. The production sector of the economy is subject to exogenous productivity change. However, technical change is of the "embodied" kind, which means that a specific capital investment has been made to profit from any productivity increase. The frontier productivity A_t grows at the exogenous rate γ:

$$A_{t+1} = (1+\gamma)A_t.$$

Capital that is first used at time t uses the frontier technology of time t; future productivity increases do not affect output derived from this capital. Capital can be used for two periods, and then depreciates completely. Thus, at time t two capital vintages K_{t-1} and K_t are in use, where the index denotes the time of first usage, and the two production functions used at time t are:

$$Y_t^{t-1} = A_{t-1}K_{t-1}^{\alpha}(L_t^{t-1})^{1-\alpha},$$

$$Y_t^{t} = A_t K_t^{\alpha}(L_t^{t})^{1-\alpha},$$

Here L_t^{t-1} and L_t^t are the amounts of labor used with capital of vintage $t-1$ and t, respectively. Labor can be allocated freely between the two vintages. The feasibility condition for the goods market is:

$$C_t + K_{t+1} = Y_t^{t-1} + Y_t^t.$$

1. Provide a recursive formulation for the planning problem solved by a benevolent social planner in this economy. **(5 points)**

2. Provide the first-order and envelope conditions, and for given levels of capital, derive the optimal allocation of labor across the different vintages. What is the balanced growth rate of the economy? **(5 points)**

Question 5

Consider a version of the Ramsey problem of optimal taxation in which the government is choosing a sequence of taxes not from today onwards, but from tomorrow onwards. The idea here is that taxes can only be put in place with some delay. Assume that first period taxes on capital, τ_{k0} and labor τ_{l0} are given exogenously. Assume that the model is otherwise standard. In particular, assume that the economy has the following features:

Government's budget constraint:

$$g_t + (1 + r_t)b_t = b_{t+1} + \tau_{l,t}w_t l_t + \tau_{k,t}q_t k_t.$$

Representative Household's problem

$$\begin{aligned} & \max\ E \sum_t \beta^t u(c_t, l_t) \\ & \text{subject to} \\ (1 - \tau_{l,t})w_t l_t + (1 - \tau_{k,t})q_t k_t + (1 + r_t)b_t \quad = \quad & c_t + k_{t+1} - (1 - \delta)k_t + b_{t+1} \\ & k_{t+1} \geq 0 \text{ and } k_0, b_0, r_0 \text{ given} \end{aligned}$$

Representative Firm's Problem:

$$\max\ [f(k_t, l_t) - q_t k_t - w_t l_t]$$

Resource Constraint:

$$c_t + k_{t+1} - (1 - \delta)k_t + g_t = f(k_t, l_t)$$

(Note that for simplicity we assume that depreciation isn't deductible.)

1. Construct the set of constraints that any primal approach to the government's problem must satisfy. Be sure to show that any allocation $\{c_t, l_t, k_{t+1}\}$ that satisfies these constraints can be mapped into a policy for the government and associated competitive equilibrium. **(3 points)**

2. Set up the Lagrangian that characterizes the solution to the Ramsey problem, and derive the associated first-order conditions. What can you say about taxes in the long-run if this economy converges to a steady state in which government expenditures was constant at g? **(3 points)**

3. Now assume that the households in our economy have the option of moving their capital abroad, where it will earn return $\bar{R}$ per period where $1 - \delta < \bar{R} < \beta^{-1}$, and that this return cannot be taxed. How would this change the solution to the problem in part (B)? Would it change the long-run tax rate on capital? **(4 points)**

Question 6

Consider the following *static* model of price adjustment with monopolistic competition. There is a single final good, and a measure 1 continuum of intermediate goods. There is a final goods sector, which uses the intermediates to produce the final good according to the technology

$$Y = \left[\int_0^1 (y(i))^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

The demand for the final good is given by $Y = M/P$, where $M > 0$ is a constant and P is the price of the final good.

The intermediate producers are monopolists. Their technology is given by the production function $y = (An)^{\alpha}$, with $\alpha \leq 1$, where n is a labor input which is hired at a constant rate $W > 0$. A can take on one of two values, $\underline{A}$ and $\bar{A}$, each with probability $1/2$, and with $\bar{A} > \underline{A}$ A fraction $\lambda \in [0, 1]$ of firms can set its prices after A is realized (flexible prices), while a fraction $1 - \lambda$ must set its price before A is known (pre-set prices).

1. Determine the demand for intermediates, as a function of their own prices, the final goods price and the total demand. Use this to derive a formula for the final goods price P. **(3 points)**

2. Characterize the firm's profit functions, and derive the first-order conditions for optimal prices, for both the pre-set and the flexible price-setters. **(3 points)**

3. What can you say about the effects of a positive TFP shock (high A) for the resulting level of prices, output, and employment in this model, (i) with exclusively preset prices, i.e. $\lambda = 0$, (ii) with exclusively flexible prices, i.e. $\lambda = 1$, and (iii) for an economy which includes both, i.e. $\lambda \in (0, 1)$? **(4 points)**