Instructions:

- Answer any 5 out of 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. DO NOT answer all questions.
- Use separate bluebook for each part.
- You have four hours to complete the exam.
- Calculators and other electronic devices are not allowed.
1. **Monopoly and Product Quality**

A type \( t \) consumer is willing to pay \( B_t(q) = t - 2q \) for a product of quality \( q \). (Consumers buy either one unit or stay out of the market.) The market price of quality \( q \) is \( P(q) \).

(a) Define \( q(t) \) to be the optimal choice of a type \( t \) consumer. Show that \( q(t) \) is a non-decreasing function.

Henceforth assume that the unit cost of producing quality level \( q \) is \( q^2 \) and that all quality levels are produced by a single monopolist. Suppose also that there are equal numbers of each type. The monopolist knows the distribution of types but the type of each consumer is private information.

(b) Explain carefully why, for profits to be maximized, “local downward constraints” must be binding.

(c) Suppose that there are just two types \( t \in T = \{\alpha t, \tau\} \). Solve for the profit maximizing qualities and prices.

(d) For what values of \( \alpha \) will both quality levels be produced?

(e) Suppose next that \( T = \{4, 8, 12\} \). Again there is an equal number of each type. What quality levels will be produced? Explain carefully.
2) Consider the following variant of Klette and Kortum. There is a fixed set \([0, 1]\) of consumers of total unit mass. Consumers are matched to firms, but each firm can serve at most two consumers. A consumer that is matched to a firm gives the firm a flow of one unit of profits. Firms that have one customer can invest to try to steal another customer from other firms according to the following technology: to generate a Poisson arrival \(\lambda\) of stealing a customer they must pay a flow cost \(c(\lambda)\) that is strictly increasing and strictly convex. In turn, outsiders may enter stealing a customer to one of the incumbents at a cost \(F\). A firm that loses all of its consumers exits. Let \(\varepsilon\) represent the equilibrium flow of entry of new firms. Denote by \(\mu_1\) and \(\mu_2\) the steady state measure of firms with one and two customers.

1. Let \(\eta\) denote the equilibrium Poisson arrival rate for the risk of losing a customer (per unit customer). Write down an equation expressing \(\eta\) as a function of \(\varepsilon, \lambda\) and \(\mu_1\).

2. Write down the Bellman equations for a firm with one and with two customers. Define a stationary equilibrium.

3. Suppose \(c(\lambda) = \lambda^2/2\). Suppose \(F\) is sufficiently low so that in equilibrium \(\varepsilon > 0\). Show that there is a unique equilibrium (Hint: use \(v_1 = F\) and the Bellman equations you defined above, noting that \(\lambda = (\nu_2 - \nu_1)\)).

4. Express the invariant measures \(\mu_1\) and \(\mu_2\) as a function of \(\lambda\) and \(\eta\) in the stationary equilibrium. (Hint: note that since the total number of customers is normalized to a unit mass, \(\mu_1 + 2\mu_2 = 1\).)

5. Letting \(R = \mu_1 c(\lambda)\) denote total investment in R&D by incumbent firms, explain why changes in \(F\) may have an ambiguous effect on \(R\).

3) Industry equilibrium. Consider the following industry equilibrium problem. There is a continuum of identical potential entrants and cost of entry \(c_0 > 0\). All firms are identical and produce according to a cost function \(c(g)\) which is strictly convex with \(c(0) > 0\). Demand in the industry is given by the inverse demand function \(p_t = D(Q_t / \gamma_t)\), where \(Q_t\) corresponds to total industry output and \(\gamma_t\) is a demand shift factor, where \(D\) is strictly decreasing and continuous. Assume that all potential entrants observe the current realization of \(\gamma_t\) prior to entry, and can start production the same period they enter into the industry. Assume also that \(\gamma_t\) evolves according to the following stochastic process: (i) up to some random time \(T\), \(\gamma_t\) grows at a constant rate \(g\); (ii) after time \(T\), it falls at a constant rate \(d\); (iii) the random time \(T\) is determined as follows: while the industry is in the growth stage, there is a constant probability \(\rho\) of switching to the decaying stage. This switch, when it occurs, is permanent.

1. Define a competitive industry equilibrium for this industry. Argue briefly -and precisely- why there exists a unique equilibrium.
2. Characterize the equilibrium the best you can: describe what happens to price and the number of firms in the growing and decaying stages. Provide a detailed answer. Hint: i) Recall The Shakeout paper for the growing phase; ii) Note that price must fall in the decaying stage.

3. Empirical evidence on the growth of industries shows that most of the growth in employment in expanding industries is explained by net entry of firms, while only 50% of the reduction in total employment in contracting industries is explained by net exit. Briefly explain why this model may provide an answer to this problem. What other testable implications does this model have?

4. Suppose now that exiting firms get a scrap value $0 < \phi \leq c_e$ when they exit. How does the equilibrium change as $\phi$ increases?

5. Conjecture, as precisely as you can, how the evolution of number of firms and size in this model would be affected by the presence of heterogeneity, i.e. if the cost of firms were affected by idiosyncratic and persistent shocks.
4) Consider modelling consumers' decisions when to buy a new car. Assume that:
- There is only one type of car with price $p$, which is constant over time.
- Consumers own at most one car at a time.
- Consumers choose at the beginning of each year whether to buy a new car.
- Consumers discount the infinite future at rate beta.

Single period utility is: $U(A) + G$

where
- $U(A)$ is the utility derived from owning a car of age $A$. $U(A)$ declines in $A$ because of increased probability of breakdown/repair costs.
- $G$ is income spent on all other goods $= Y(i) - p$ if buy a new car
  $= Y(i)$ if don't buy a new car.

where $Y(i)$ is consumer $i$'s per-period income (assume $Y(i)$ is constant over time).

Write down the dynamic decision problem (Bellman Equation) that tells us the PDV of future utilities for a consumer. Be sure to specify what the state variable(s) are, how they evolve over time, and exactly what any expectations in the equation are over.

5) Describe the Berry/BLP "inversion" in the context of 1) the logit model, 2) the nested logit model, and 3) a random coefficients model.

Why is this invertibility property important? For example, what would change if these models were _not_ invertible? How would estimation change? Would you need to make additional assumptions?

6) Consider the homogeneous product demand curve:

$Q(t) = \beta_0 + \beta_1 P(t) + \beta_2 X(t) + \beta_3 Z(t) + \beta_4 Z(t) P(t) + \varepsilon(t)$

where $t$ indexes market, $Q(t)$ and $P(t)$ are observed quantity and price, and $X(t)$ and $Z(t)$ are observed exogenous variables that affect demand. $\varepsilon(t)$ is an unobserved demand shock.

Assume that firms are identical, and have marginal cost functions of:

$MC(t) = \gamma_0 + \gamma_1 Q(t) + \gamma_2 W(t) + \eta(t)$

where $W(t)$ is an observed cost shifter and $\eta(t)$ is a cost shock.

a) Derive the industry supply relationship for this market as a function of theta, the "conjectural variations" parameter.

b) Describe precisely the interpretation of theta.

c) Under what conditions on the demand parameters (beta) is theta(t) identified? In other words, for what values of beta is theta identified, and for what values is theta not identified? Prove (with a similar degree of rigor as used in class) that this is the case.