# UCLA

# Department of Economics

# Ph. D. Preliminary Exam

# Micro-Economic Theory

(SPRING 2017)

**Instructions**:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

## 1. Equilibrium with Uncertainty

There are two individuals living on the opposite side of an island: one on the eastern side and another on the western side. They only consume a particular type of banana that grows naturally in this island. Their endowments of banana are affected by the weather condition. There are two possible states of the world $s_1$ and $s_2$, which are equally likely. At state $s_1$, consumer $E$ (who lives on the eastern side) is endowed with 30 bananas and consumer $W$ is endowed with 10 bananas. This is a high (total) precipitation state with relatively more rain on the eastern side. At state $s_2$, consumer $E$ is endowed with 5 bananas and consumer $W$ is endowed with 15 bananas. This is a low precipitation state with relatively more rain on the western side. Each individual is an expected utility maximizer. Their expected utility is given by $0.5u_i(x_{s_1,i}) + 0.5u_i(x_{s_2,i}), i = E, W$, where $x_{s,i}$ is consumer $i$'s consumption of banana in state $s$. Assume that $u_i(x) = \log x$ for $i = E, H$ except for question (c).

(a) Define Pareto efficient allocation and characterize the set of Pareto efficient allocations in this economy.

(b) Find an Arrow-Debreu equilibrium and verify that the equilibrium price of "one banana at $s_2$" is higher than the equilibrium price of "one banana at state $s_1$".

(c) Suppose that $u_i$ is just a differentiable, strictly increasing and strictly concave function. Prove that the property shown in (b) (about the equilibrium prices) still holds. A graphical answer using Edgeworth box is acceptable (as long as your presentation/explanation is clear enough).

For the next two questions, assume that the following two real assets are available for trading in the first period (before the weather/state realizes and bananas are consumed in the 2nd period). Asset A is a safe asset: the owner of 1 unit of Asset A is entitled to receive 1 banana independent of the state realization. Asset B is like an insurance: the owner of 1 unit of Asset B is entitled to receive 3 bananas in the low precipitation state $s_2$ but nothing in the high precipitation state $s_1$. Both consumers can buy or sell these two assets (and only these two assets) freely in the first period.

(d) Explain why the equilibrium consumption in any Radner equilibrium must be the same as the equilibrium consumption in the Arrow-Debreu equilibrium in (b).

(e) Find a Radner equilibrium (which consists of the equilibrium consumption/price of banana in each state in the 2nd period, and the equilibrium asset holding and asset prices in the 1st period).

**2. Demand Functions with Expected Utility**

There is a consumer who faces uncertainty and is trying to come up with his/her consumption plan. There are three states of nature: $s_1, s_2, s_3$ and there is only one good for consumption. A consumer's consumption plan is denoted by $x = (x_1, x_2, x_3)$, where $x_k \in R$ is the consumer's consumption of the good at state $s_k$ for $k = 1, 2, 3$. We assume that this consumer's preference over $x \in R^3_+$ can be represented in the following expected utility form: $U(x) = \pi_1 u(x_1) + \pi_2 u(x_2) + \pi_3 u(x_3)$, where $\pi_k > 0$ is the (subjective) probability of state $s_k$. Assume that $u$ is continuously differentiable, $u' > 0$, $u'' < 0$, and satisfies $\lim_{x_k \downarrow 0} u'(x_k) = \infty$ for $k = 1, 2, 3$. The consumer can buy any state-contingent good $x_k$ at price $p_k > 0$ for $k = 1, 2, 3$. Answer the following questions.

(a) Write down the (expected) utility maximization problem given price $p = (p_1, p_2, p_3)$ and initial wealth $w > 0$. Also write down the conditions to characterize this consumer's (Walrasian) demand function.

(b) Assume only for this question that $\pi_1 = \pi_2 = \pi_3 = \frac{1}{3}$, and $u(x_k) = \sqrt{x_k - a}$, where $a > 0$ is some exogenous parameter ($u(x_k) = -\infty$ if $x_k < a$). Also assume that the consumer's wealth $w$ satisfies $w > (p_1 + p_2 + p_3)a$. Derive the Walrasian demand function explicitly. Is each (state-contingent) good a normal good (i.e. $\frac{x_k(p,w)}{\partial w} > 0$, where $h_k$ is the Walrasian demand function of $x_k$) or an inferior good(i.e. $\frac{x_k(p,w)}{\partial w} < 0$)? Are any pair of goods substitutes (i.e. $\frac{\partial h_k(p,u)}{\partial p_j} \geqq 0$, where $h_k$ is the Hicksian demand function of $x_k$) or complements (i.e. $\frac{\partial h_k(p,u)}{\partial p_j} \leqq 0$)? Discuss.

(c) Show that, for any $j$, $\frac{\partial h_k(p,u)}{\partial p_j}$ has the same sign for all $k \neq j$ (**Hint:** Examine the first order conditions carefully. You can assume that everything is nicely differentiable).

(d) Is each good a normal good or an inferior good? Are any pair of goods are substitutes or complements? Discuss.

**3. Pig in a Poke**

A worker is producing and selling a widget of low or high quality $\theta = L, H$ where we normalize $L = 0$ and $H = 1$. Producing low quality is costless to the worker while high quality costs him $c > 0$, which is distributed with (smooth) cdf $F(c)$ and known only to the worker. A competitive market of buyers observes an imperfect signal $s = \ell, h$ of the quality, where $\Pr(\ell|L) = 1$ and $\Pr(h|H) = \pi < 1$; that is upon observing $h$, buyers know that $\theta = H$, but upon observing $\ell$ uncertainty remains. The timing is as follows. First the worker chooses $\theta$; we denote his strategy by $x(c) = \theta$. Next buyers observe $s$. Finally, buyers bid the price up to the expected value $p_s = E[\theta|s, x(\cdot)]$, conditional on the signal $s$ and the worker's equilibrium strategy $x(\cdot)$. Worker $c$'s utility from producing quality $\theta$ and selling it for $p$ equals $p - c\theta$.

(a) First assume that the signal is uninformative, i.e. $\pi = \Pr(h|H) = 0$. What is the equilibrium of this game?

From now on assume $\pi > 0$.

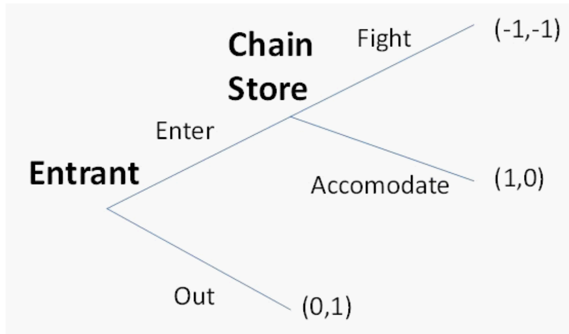
(b) Assume that low-cost types $c \leq c^*$ produce high quality, and high-cost types $c > c^*$ produce low quality for some arbitrary threshold $c^*$ (we simply denote this strategy $x(\cdot)$ by $c^*$). What is buyers' expectation of quality, $p_0 = E[\theta|c^*]$, i.e. before observing the signal $s$? Use Bayes' rule to calculate the resulting prices $p_s = E[\theta|s, c^*]$ after signals $s = \ell, h$! Show that $p_\ell$ increases in $c^*$! Discuss!

(c) Argue that in equilibrium, the threshold $c^*$ must satisfy

$$c^* = \pi(E[\theta|h, c^*] - E[\theta|\ell, c^*]), \tag{1}$$

and argue that this equation defines a unique value for $c^*$!

(d) Now consider an increase in the signal quality $\pi = \Pr(h|H)$. Does the threshold $c^*$ rise or fall? Discuss!




## 4 Chain Store Game with Short Memory:

A chain store is facing an infinite sequence of potential, short-lived entrants $t = 1, 2, 3....$ The period-$t$ stage game is given by the game tree above.[1]

Entrant-$t$ maximizes his period-$t$ payoff; the chain store maximizes long-term payoffs, discounted at rate $\delta < 1$.

(a) What is the SPE of the stage game?

Now consider the dynamic game.

(b) Assume first that the game has perfect information and consider the following "grim-trigger" strategy profile: If the chain store has always fought entry in the past (or no entrant has ever entered), then the entrant stays out and (if he enters) the chain store fights; otherwise the entrant enters and the chain store accommodates. For which levels of $\delta$ is this an equilibrium?

(c) Now assume that entrant-$t$ observes only the outcome, i.e. O,F, or A, of period $t-1$ (and assume for convenience that the outcome in period $t = 0$ was O). Also assume that $\delta \neq 1/2$. Consider the following "short memory grim-trigger" strategy profile: If period $t-1$'s outcome was O or F, entrant-$t$ stays out and (if he enters) the chain store fights; if period $t-1$'s outcome was A, entrant-$t$ enters and the chain store accommodates.[2] Show that this strategy profile is not a SPE! Discuss!

(d) Assume that the chain store is sufficiently patient, that is $\delta > 1/2$, and solve for a mixed strategy equilibrium of the following kind: If last period's outcome was O or F, the entrant stays out and (if he enters) the chain store fights with probability $p$; if last period's outcome was A, the entrant enters with probability $q$ and the chain store fights with probability $p$.

[1] As usual, the first payoff is for the entrant and the second for the chain store, for instance for the accommodate outcome, 1 for the entrant and 0 for the chain store.

[2] Thus, on-path all entrants stay out; but off-path, after an A outcome, all subsequent entrants enter and the chain store accommodates.

**5. Spencian Signaling**

A type $\theta$ worker has a marginal product (value to competing firms) of $m(\theta) = 2 + \theta$. A type $\theta$'s payoff if he chooses not to signal is $U_O(\theta) = 1$. The support of the distribution of types is $[0, \beta]$ and the c.d.f. is $F(\theta) \in \mathbb{C}^1$. In country 1 the cost of signaling at level $z$ is $C_1(\theta, z) = \frac{z}{\theta^{\frac{1}{2}}}$. In country 2 the cost of signaling at level $y$ is $C_2(\theta, y) = \frac{y}{2+\theta}$.

(a) Explain why the PBE constraint determines the separating PBE payoff of the lowest type.

(b) In country 1 show that a separating PBE payoff function is a solution of the following differential equation

$$\frac{d}{d\theta}[\theta^{\frac{1}{2}} U(\theta)] = \frac{1}{2}\theta^{-\frac{1}{2}}(2+\theta) = \theta^{-\frac{1}{2}} + \frac{1}{2}\theta^{\frac{1}{2}}.$$

Hence solve for the IC mapping $K_1(\theta, u)$ (i.e. the family of solutions to the ODE). Solve also for the differential equation in country 2 and hence the IC mapping $K_2(\theta, u)$.

(c) Show that the equilibrium payoff function in country 1 is $U_1(\theta) = 2 + \frac{1}{3}\theta$. Solve also for the PBE payoffs in country 2.

(d) Which signaling technology is better (i) for low types (ii) for sufficiently high types?

(e) Suppose that workers can move between two countries freely, so have access to both signaling technologies. Solve for the set of types that signal in country 2 in the Pareto-dominant separating PBE.

(f) How would your answer to part (e) change if instead $U_O(\theta) = 2 + \frac{1}{3}\theta$?

**6. Indirect Price Discrimination**

A firm has a cost function $C(q) = c_0 + q$. If a type $\theta$ buyer purchases $q$ units and pays $r$, then his payoff is

$$u(\theta, q, r) = B(\theta, q) - r = \int_0^q p(\theta, x)\, dx - r,$$

where $p(\theta, q) = 2 + \theta - q$.

The distribution of types has support $\Theta = [0, 1]$. The p.d.f and c.d.f. and hazard rate $h(\theta) = \frac{f(\theta)}{1-F(\theta)}$ are given below where $b \in (0, 1)$.

$$f(\theta) = \begin{cases} \frac{1}{2b}, & 0 \leq \theta < b \\ \frac{1}{2(1-b)}, & b \leq \theta \leq 1 \end{cases} \quad F(\theta) = \begin{cases} \frac{\theta}{2b}, & 0 \leq \theta < b \\ \frac{1}{2} + \frac{\theta - b}{2(1-b)}, & b \leq \theta \leq 1 \end{cases} \quad h(\theta) = \begin{cases} \frac{1}{2b-\theta}, & 0 \leq \theta < b \\ \frac{1}{1-\theta}, & b \leq \theta \leq 1 \end{cases}$$

(a) Confirm that the Single Crossing Property holds. Then either prove or explain clearly with a diagram, why a necessary condition for incentive compatibility is that the outcome mapping $\{q(\theta), r(\theta)\}_{\theta \in \Theta}$ must be increasing.

(b) For any monotonic allocation $\{q(\theta)\}_{\theta \in \Theta}$, show that the designer's maximum expected revenue is

$$E[r(\theta)] = E[R_D^V(\theta, q)]$$

where

$$R_D^V(\theta, q) = B(\theta, q) - \frac{1}{h(\theta)} \frac{\partial B(\theta, q)}{\partial \theta}.$$

(c) If $b = \frac{1}{2}$ (the uniform case) solve for the profit maximizing allocation.

(d) The firm implements the selling scheme by selling a "q-pack" for each $q \in [q(0), q(1)]$. Solve for the price of a q-pack if $b = \frac{1}{2}$.

(e) Next suppose that $\frac{1}{2} < b < 1$. Show that the hazard rate is increasing. Solve for the new profit maximizing allocation $q(\theta)$ if $b = \frac{3}{4}$. Discuss how this might be implemented.