Answers

Answer for Q1

(a) (2 pts: 0.5 pts. for the definition and 1.5 pts. for its characterization) The definition of PE is standard. There may be many ways to characterize the set of PE allocations. But whichever way is taken, eventually the following system of equations is obtained.

$$\begin{aligned}\frac{x_{s_2,E,}}{x_{s_1,E}} &= \frac{x_{s_2,W}}{x_{s_1,W}} \text{ (MRS for the state-contingent goods are equalized)}\\ x_{s_1,E}+x_{s_1,W} &= 40 \text{ (resource constraint for banana at } s_1)\\ x_{s_2,E}+x_{s_2,W} &= 20 \text{ (resource constraint for banana at } s_2)\end{aligned}$$

Simplifying this, we obtain the following set of PE allocations: $(x_E, x_W) \in \{(40\alpha, 20\alpha), (40(1-\alpha), 20(1-\alpha)) \,|\alpha \in [0,1]\}$.

(b) (2 pts.) By the first welfare theorem, the Arrow-Debreu equilibrium allocation must be Pareto-efficient. Hence the equilibrium price ratio $\frac{p_1^*}{p_2^*}$, which is equal to MRS $\frac{x^*_{s_2,E}}{x^*_{s_1,E}} = \frac{x^*_{s_2,W}}{x^*_{s_1,W}}$, must be $\frac{1}{2}$. Thus a banana at s_2 must be twice more expensive as a banana at s_1. Given this equilibrium price ratio, it is easy to derive the following optimal consumption vectors: $\left(x^*_{s_1,E}, x^*_{s_2,E}\right) = \left(x^*_{s_1,W}, x^*_{s_2,W}\right) = (20, 10)$, which of course satisfy the market clearing conditions.

(c) (2 pts.) For general u_i, we have $\frac{u_i'\left(x^*_{s_1,i}\right)}{u_i'\left(x^*_{s_2,i}\right)} = \frac{p_1^*}{p_2^*}$ in equilibrium for $i = E, W$. If $\frac{p_1^*}{p_2^*} = 1$, then the consumers would obtain a full insurance, i.e. $\frac{x^*_{s_2,i}}{x^*_{s_1,i}} = 1$ for $i = E, W$. If $\frac{p_1^*}{p_2^*} > 1$, then $\frac{x^*_{s_2,i}}{x^*_{s_1,i}}$ would be even strictly larger than 1 for $i = E, W$. Hence the total consumption of banana at state s_2 is at least as large as the total consumption of banana at state s_1 if $\frac{p_1^*}{p_2^*} \geqq 1$. But then the markets would not clear as the total supply of banana is larger at s_1, so this is a contradiction. Hence the equilibrium price ratio $\frac{p_1^*}{p_2^*}$ must be strictly less than 1.

(d) (1.5 pts.) This is because....

- The Arrow-Debreu equilibrium allocation is unique in this economy with log utility function.
- The market is complete with two independent assets for two states.
- The set of Radner equilibrium allocations coincides with the set of A-D equilibrium allocations in a complete market.

(e) (2.5 pts.) Since this is an economy with one good, there will be no trade in period 2 given any strictly positive price of banana in each state. Let's normalize the price of banana in each state to 1. In A-D equilibrium, consumer E exchanges 10 bananas at s_1 for 5 bananas at s_2. Given the equivalence between the Radner equilibrium and the A-D equilibrium, consumer E must buy 5 units of Asset B and sells 10 units of Asset A (this is the only way as this is a complete market case). Of course consumer W must sell 5 units of Asset B and buy 10 units of Asset A. This asset trading in period 1 is feasible if $\frac{q_A^*}{q_B^*} = \frac{1}{2}$, where q_k^* is the equilibrium price of asset $k = A, B$. So one possible Radner equilibrium would be $\left(x_E^*, x_W^*, p_{s_1}^*, p_{s_2}^*, q_A^*, q_B^*\right) = \left(\left(20, 10\right), \left(20, 10\right), 1, 1, 1, 2\right)$

Answer for Q2

(a) (2 pts: 1 pt for writing down the problem correctly and 1 pt for the conditions) The maximization problem is

$$\begin{gathered}\max_{x_k \geq 0} \pi_1 u(x_1) + \pi_2 u(x_2) + \pi_3 u(x_3) \\ s.t.\ p_1 x_1 + p_2 x_2 + p_3 x_3 \leq w\end{gathered}$$

The optimality condition is

$$\begin{aligned}\pi_k u'(x_k) - \lambda p_k &= 0 \text{ for } k = 1, 2, 3 \\ p_1 x_1 + p_2 x_2 + p_3 x_3 &= w\end{aligned}$$

(b) (3.5 pts.) Define $x'_k = x_k - a$ and $w' = w - (p_1 + p_2 + p_3) a > 0$. Note that the objective function can be transformed into a CES function: $\left(\sqrt{x'_1} + \sqrt{x'_2} + \sqrt{x'_3}\right)^2$ and the budget constraint becomes $p_1 x'_1 + p_2 x'_2 + p_3 x'_3 \leq w'$. So the solution is

$$x'_k(p, w') = \frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}} w'.$$

Hence we have

$$x_k(p, w) = a + \frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}} (w - (p_1 + p_2 + p_3) a)$$

It is clearly a normal good: $\frac{\partial x_k(p,w)}{\partial w} > 0$ for any k. Note that $\frac{\partial h_k(p,u)}{\partial p_j} = \frac{\partial x_k(p,w)}{\partial p_j} + \frac{\partial x_k(p,w)}{\partial w} x_j(p, w) > 0$ (Slutsky equation) for any $j \neq k$. Hence

$$\begin{aligned}\frac{\partial h_k(p,u)}{\partial p_j} &= \frac{\partial}{\partial p_j}\left(\frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}}\right) w' - a \left(\frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}}\right) + \frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}}\left(a + \frac{\left(\frac{1}{p_j}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}} w'\right) \\ &= \frac{\partial}{\partial p_j}\left(\frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}}\right) w' + \left(\frac{\left(\frac{1}{p_k}\right)^2}{\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3}}\right)^2 w' \\ &> 0.\end{aligned}$$

So any pair of goods are substitutes, not complements.

(c) (1.5 pts.) Let $p_k - \lambda \pi_k u'(x_k) = 0$ be the first order condition for x_k for the cost minimization problem. Differentiating this with respect to p_j, $j \neq k$, we obtain $\frac{\partial h_k(p,u)}{\partial p_j} = -\frac{u'}{u''\lambda}\frac{\partial \lambda}{\partial p_j}$. This implies that, for each j, $\frac{\partial h_k(p,u)}{\partial p_j}$ has the same sign for all $k \neq j$.

(d) (3 pts: 1.5 pts each)

- (Normal good): The first order condition is $\pi_k u'(x_k) - \lambda p_k = 0$ for $k = 1, 2, 3$. If λ increases, then every consumption must decreases (for fixed p). Then, as w increases, λ must decrease and every consumption must increase so that the budget constraint is binding. Hence every good is a normal good.
- (Substitutes): Suppose that $\frac{\partial h_k(p,u)}{\partial p_j} < 0$ for some j and $k \neq j$. Then $\frac{\partial h_k(p,u)}{\partial p_j} < 0$ for any $k \neq j$ by (c). We also know that $\frac{\partial h_j(p,u)}{\partial p_j}$ is not positive because S is negative semi-definite (cost function is concave). Then this is a contradiction because $h_k(p, u)$ cannot decrease for every k as p_j increases. Hence it must be the case that $\frac{\partial h_k(p,u)}{\partial p_j} \geq 0$ for every j and $k \neq j$. That is, any pair of the goods must be substitutes for an additive utility function.

Answer for Q3

1. (a) (2 points) Since the signal is ℓ for either quality level, and hence the price $p = E[\theta|\ell, \widetilde{x}(\cdot)]$ does not depend on actual quality (but only on buyers' beliefs over quality) the worker chooses low quality for any $c > 0$. Thus, $p_\ell = p_h = 0$.

   From now on assume $\pi > 0$.

   (b) (3 points) Before observing s, buyers believe that quality is high with probability

$$p_0 = F(c^*).$$

   After observing $s = H$, they know that quality is high,

$$p_h = E[\theta|h, c^*] = \frac{p_0 \Pr(h|H)}{p_0 \Pr(h|H) + (1-p_0)\underbrace{\Pr(h|L)}_{=0}} = 1.$$

   After observing $s = L$, Bayes' rule implies

$$\begin{aligned} p_\ell &= E[\theta|\ell, c^*] = \frac{p_0 \Pr(\ell|H)}{p_0 \Pr(\ell|H) + (1-p_0)\Pr(\ell|L)} = \\ &= \frac{F(c^*)(1-\pi)}{F(c^*)(1-\pi) + (1-F(c^*))} = \frac{(1-\pi)F(c^*)}{1-\pi F(c^*)} = \frac{1-\pi}{\frac{1}{F(c^*)} - \pi}, \end{aligned}$$

   which increases in c^*. Intuitively, the higher the prior belief of high quality, the higher the posterior belief after signal ℓ, since the market is now attributing this bad signal more to bad luck and less to low quality.

   (c) (3 points) The benefit of producing high quality is that it increases the probability of a high signal from 0 to π, and a high signal raises the price by $E[\theta|h, c^*] - E[\theta|\ell, c^*]$. Thus the benefit of a high signal is given by the RHS of (1). In equilibrium, this must equal the cost of producing high quality for the threshold type c^*, the LHS.

   Since p_ℓ increases in c^* while p_h is constant, equal to 1, the RHS decreases in c^*. The LHS clearly increases in c^*. Thus they cross at most once, and hence the equilibrium threshold c^* is unique.

   (d) (2 points) The RHS of (1)

$$\pi(E[\theta|h, c^*] - E[\theta|\ell, c^*]) = \pi(1 - \frac{(1-\pi)F(c^*)}{1-\pi F(c^*)}) = \pi \frac{1-F(c^*)}{1-\pi F(c^*)}$$

   rises in π; intuitively, a rise in π increases both the likelihood (π) that producing high quality results in the h signal, and the informativeness (and hence the value) of this signal $p_h - p_\ell$. To restore equality in (1), c^* has to rise, increasing the LHS and decreasing the RHS. Intuitively, when signal quality rises, more types (with higher costs) can be incentivized to produce high quality.

Answer for Q4

1. (a) (1 point) By backward induction, the chain store will accomodate entry, and hence the entrant enters.

   (b) (2 points) The entrants' strategy is clearly a best response to the chain store. As for the chain store, the critical question is whether he is willing to fight entry today (cost $u_F = -1$), to secure a monopoly position for the future (benefit $u_O = 1$): $(1-\delta)u_F \leq \delta u_O$, i.e. $\delta \geq u_F/(u_O + u_F) = 1/2$.

   Now assume that entrant-t observes only the outcome, i.e. O,F, or A, of period $t-1$ (and assume for convenience that the outcome in period $t = 0$ was O). Also assume that $\delta \neq 1/2$.

   (c) (3 points) The entrants' strategy is clearly a best response to the chain store. As for the chain store:

   - If last period's outcome was O or F, and the entrant (unexpectedly) enters the condition for the chain store to follow his strategy and fight is as in part (a): $(1-\delta)u_F \leq \delta u_O$, or $\delta \geq u_F/(u_O + u_F) = 1/2$. The chain store must be sufficiently patient to "defend his reputation by fighting entry".
   - But if last period's outcome was A, and the entrant enters, the condition for the chain store to follow is strategy and accomodate is the reverse: $(1-\delta)u_F \geq \delta u_O$, or $\delta \leq u_F/(u_O + u_F) = 1/2$. The chain store must be sufficiently impatient to "accept the punishment phase rather than fighting to restore his reputation".
   - These conditions cannot both hold since we ruled out the knife-edge case $\delta = u_F/(u_O + u_F) = 1/2$.

   (d) (4 points) For the entrant (after any history) the expected cost of being fought (cost $v_F = -1$) must equal the expected benefit of being accomodated (benefit $v_A = 1$), i.e. $pv_F + (1-p)v_A = 0$, i.e. $p = v_A/(-v_F + v_A) = 1/2$. The chain store (after entry in period t) must be indifferent between (i) fighting in t and having entrant $t+1$ stay out, and (ii) accomodating in t, having entrant $t+1$ enter with probability q, and if so fight entry; both plans have entrants $t' \geq t+2$ stay out. Plan (i) yields utility $-u_F + \delta u_O$ (in periods t and $t+1$). Plan (ii) yields utilty $\delta(-qu_F + (1-q)u_O)$. Equating yields $u_F = \delta q(u_F + u_O)$, i.e. $q = u_F/(\delta(u_O + u_F)) = 1/(2\delta)$.

Answer to question 5

(a) 1 point

The firms know that any worker's value is at least the lowest value. Therefore $r(\alpha)\geq$ $m(\alpha)=2+\alpha=2$.

In a separating PBE every signal is chosen by 1 type. Therefore $(q(\alpha),r(a))=(q(\alpha),m(\alpha))=(q(0),2)$

Also for any $\hat{z}<q(\alpha)$, the wage offer $r(\hat{z})\geq m(\alpha)=2$. Thus $q(\alpha)$ is a best response if and only if $q(\alpha)=0$ and so $(q(\alpha),r(\alpha)=(0,2)$. Therefore $U(\alpha)=2$.

(b) 3 points

$$q(\theta)\in\arg\underset{x}{Max}\{u(\theta,q(x),r(x)=r(x)-C(\theta,q(x))=2+x-\frac{q(x)}{A(\theta)}\}$$

$$U(\theta)=2+\theta-\frac{q(\theta)}{A(\theta)}$$

Envelope Theorem

$$U'(\theta)=\frac{A'(\theta)}{A(\theta)}\frac{q(\theta)}{A(\theta)}=\frac{A'(\theta)}{A(\theta)}(2+\theta-U(\theta))$$

$$\frac{d}{d\theta}[A(\theta)U(\theta)]=A(\theta)U'(\theta)+U(\theta)A'(\theta)=A'(\theta)(2+\theta)$$

Case (i) $A(\theta)=\theta^{1/2}$, $A'(\theta)=\tfrac{1}{2}\theta^{-1/2}$

(One student observed that $C(q,0)$ is not defined at $\theta=0$. However $C(0,\theta)=0$ for all $\theta>0$. Thus the natural assumption is that the function is continuous so $C(0,0)=0$. To be complete I should have noted that.)

$$\frac{d}{d\theta}[\theta^{1/2}U(\theta)]=\tfrac{1}{2}\theta^{-1/2}(2+\theta)=\theta^{-1/2}+\tfrac{1}{2}\theta^{1/2}$$

$$\theta^{1/2}U(\theta)=2\theta^{1/2}+\tfrac{1}{3}\theta^{3/2}+k\,.$$

IC mapping

$$K_1(\theta,u)=\theta^{1/2}u-2\theta^{1/2}-\tfrac{1}{3}\theta^{3/2}$$

$$U_1(\theta)=2+\tfrac{1}{3}\theta>U_O(\theta)$$

Case (ii) $A(\theta)=2+\theta$, $A'(\theta)=1$

Method 1:

$$\frac{d}{d\theta}[(2+\theta)U(\theta)]=(2+\theta)$$

$$(2+\theta)U(\theta)=\tfrac{1}{2}(2+\theta)^2+k$$

IC mapping

$$K_2(\theta,u)=(2+\theta)u-\tfrac{1}{2}(2+\theta)^2$$

Method 2

$$\frac{d}{d\theta}[(2+\theta)U(\theta)]=(2+\theta)$$

Then

$$(2+\theta)U(\theta)=2\theta+\tfrac{1}{2}\theta^2+K_2$$

$$K_2=(2+\theta)u-2\theta-\tfrac{1}{2}\theta^2$$

(c) 2 points

$$U(\theta)=2+\tfrac{1}{3}\theta+\frac{k_1}{\theta^{1/2}}$$

Since $U(0)=2$ it follows that $k=0$. Then

$$U_1(\theta)=2+\tfrac{1}{3}\theta$$

Method 1:

$$U_2(\theta)=\tfrac{1}{2}(2+\theta)+\frac{k_2}{2+\theta}$$

Since $U(0)=2$ it follows that $k_2=2$. Then

$$U_2(\theta)=1+\tfrac{1}{2}\theta+\frac{2}{2+\theta}.$$

Method 2:

Since $U_2(0)=2$, $K_2=4$.

Then

$$(2+\theta)U_2(\theta)=4+2\theta+\tfrac{1}{2}\theta^2$$

$$U_2(\theta)=2+\tfrac{1}{2}\frac{\theta^2}{(2+\theta)}$$

(d) 1 point

Method 1	Method 2
$U_2'(0)=0 \quad U_2'(\theta)\to\frac{1}{2}$ $U_1(\theta)=2+\frac{1}{3}\theta$ Therefore 1 is better for low types And 2 is better for high types	$U_1(\theta)-U_2(\theta)=\frac{1}{3}\theta-\frac{1}{2}\frac{\theta^2}{(2+\theta)}$ $=\frac{\theta}{2+\theta}[\frac{1}{3}(2+\theta)-\frac{1}{2}\theta]$ This is positive for low types and negative for high types.

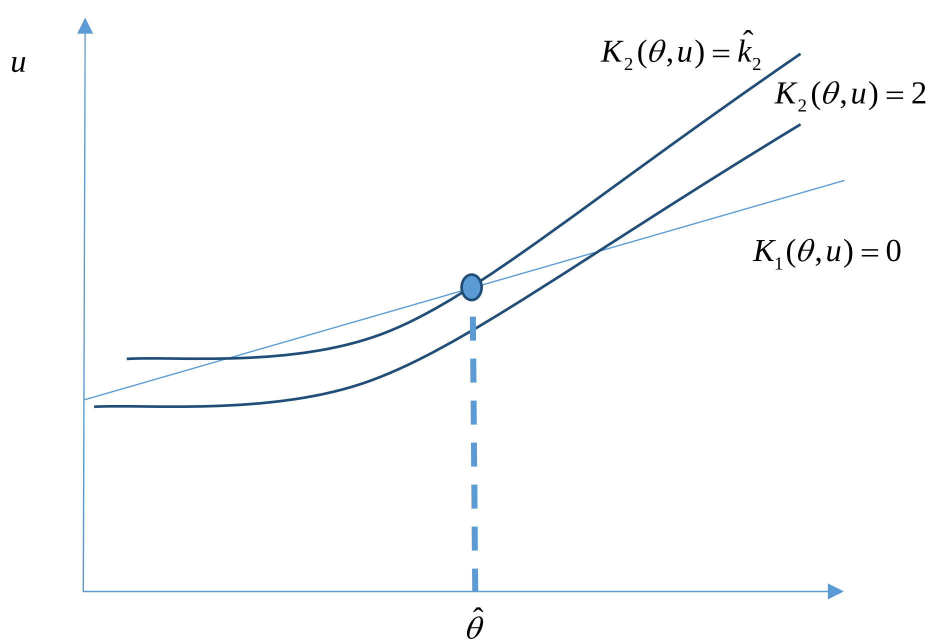
(e) 2 points

The equilibrium level sets of the IC mappings are depicted below.

The graph of $U_1(\theta)$ is the level set $K_1(\theta,u)=0$. The graph of $U_2(\theta)$ is the level set $K_2(\theta,u)=2$.

Consider $\hat{U}_2(\theta)$ on the level set depicted $K_2(\theta,u)=\hat{k}>2$

$U(\theta)=Max\{U_1(\theta),\hat{U}_2(\theta)\}$ is incentive compatible.



Along the equilibrium level set for technology 1

$U_1(\theta) = 2 + \frac{1}{3}\theta$

Along this line

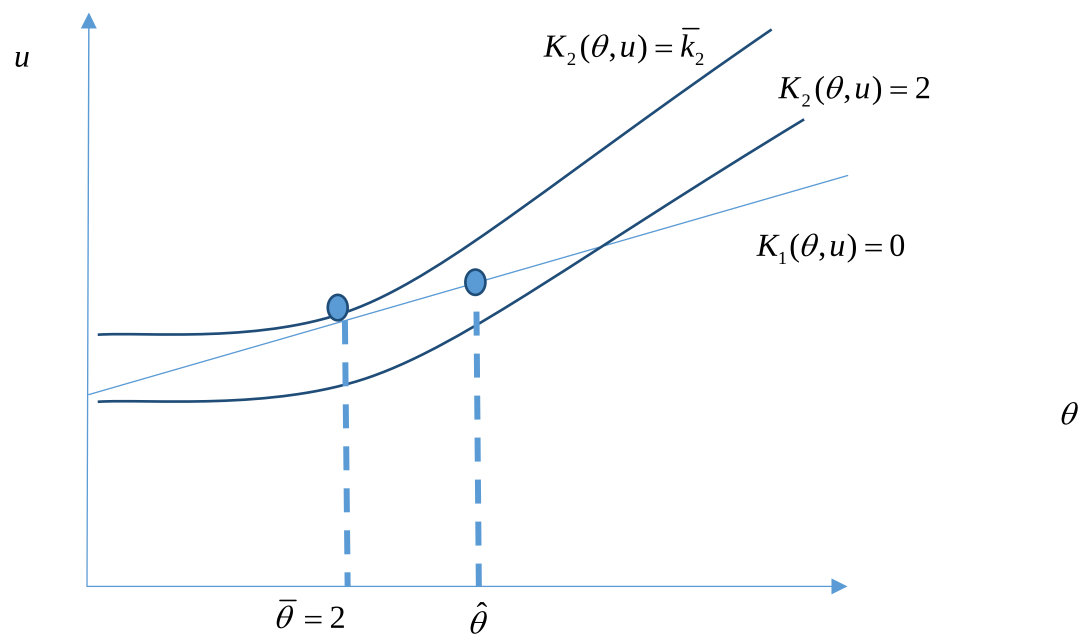
$$K_2(\theta, U_1(\theta)) = (2+\theta)(2+\tfrac{1}{3}\theta) - \tfrac{1}{2}(2+\theta)^2$$

$$\frac{d}{d\theta}K_2(\theta, U_1(\theta)) = 2 + \tfrac{1}{3}\theta + \tfrac{1}{3}(2+\theta) - (2+\theta) = \tfrac{1}{3}(2-\theta)$$

Thus the indifference maps touch at $\hat{\theta} = 2$. See the figure below.

$U(\theta) = Max\{U_1(\theta), \bar{U}_2(\theta)\}$ is incentive compatible

This is the Pareto dominant separating PBE



Then high types (($\theta \geq 2$)use technology 2 while low types us technology 1.

(f) 1 point

$U_1(\theta) = U_0(\theta)$ so equilibrium payoffs are unchanged.

PBE for types below 2 to stay out.

Bonus point

But these types are indifferent between staying out and signaling so low types signaling on [0,a] and staying out on [a,2] is also a PBE.

Answer to question 6

(a) 2 points

Along a level set $r = B(\theta,q) = k$. Hence the slope is $\dfrac{dr}{dq} = p(\theta,q)$. This is increasing in θ .

For type θ, the extra value of choosing (q,r) rather then $(\hat{q},\hat{r})$, where $q > \hat{q}$ Is

$$B(\theta,q) - B(\theta,\hat{q}) = \int_{\hat{q}}^{q} p(\theta,x)dx \ .$$

The extra cost is $r - \hat{r}$

The extra benefit is increasing in θ (this is the SCP).

If type $\hat{\theta}$ chooses $\hat{q}$ the extra value of switching to any $q > \hat{q}$ is lower than the extra cost.

$$B(\hat{\theta},q) - B(\hat{\theta},\hat{q}) = \int_{\hat{q}}^{q} p(\hat{\theta},x)dx \le r - \hat{r} \text{ for all } q > \hat{q}$$

For any lower type θ the demand price is smaller. Thus for all $q < \hat{\theta}$,

$$B(\theta,q) - B(\theta,\hat{q}) = \int_{\hat{q}}^{q} p(\theta,x)dx < r - \hat{r} \text{ for all } q > \hat{q}$$

Thus lower types will either choose $(\hat{q},\hat{r})$ or a plan with a lower q .

(b) 2 points

$U(\theta) = B(\theta,q(\theta)) - r(\theta)$.

Therefore

$$\mathrm{E}[r(\theta)] = \mathrm{E}[B(\theta,q)] - \mathrm{E}[U(\theta)]$$

$$\mathrm{E}[U(\theta)] = \int_0^1 U(\theta)f(\theta)d\theta$$

$$= U(\alpha) + \int_0^1 U'(\theta)(1 - F(\theta)d\theta$$

$$= U(\alpha) + \int_0^1 U'(\theta)(\frac{1-F(\theta)}{f(\theta)})f(\theta)d\theta$$

$$=U(\alpha)+\int_0^1 U'(\theta)\frac{1}{h(\theta)}f(\theta)d\theta$$

$$=U(\alpha)+\mathrm{E}[\frac{U'(\theta)}{h(\theta)}]$$

Therefore

$$\mathrm{E}[r(\theta)]=\mathrm{E}[B(\theta,q)]-\mathrm{E}[\frac{U'(\theta)}{h(\theta)}]-U(\alpha) \qquad (0.1)$$

We obtain the marginal information rent by appealing to the Envelope Theorem for IC allocations

For all $x\in\Theta$ buyer θ must prefer $(q(\theta),r(\theta))$ to $(q(x),r(x))$. Therefore

$$\theta\in\arg\underset{x}{Max}\{u(\theta,q(x),r(x))=B(\theta,q(x))-r(x)\}$$

$$U(\theta)=B(\theta,q(\theta))-r(\theta)$$

FOC

$$\frac{\partial B}{\partial x}(\theta,q(x))-r'(x)=p(\theta,q(x))q'(x)-r'(x)=0 \text{ at } x=\theta \text{ , i.e.}$$

$$r'(\theta)=p(\theta,q(\theta))q'(\theta)=(2+\theta-q(\theta))q'(\theta) \qquad (0.2)$$

Appealing to the Envelope Theorem

$$U'(\theta)=\frac{\partial B}{\partial\theta}(\theta,q(\theta)).$$

Therefore

$$\mathrm{E}[r(\theta)]=\mathrm{E}[B(\theta,q)]-\mathrm{E}[\frac{1}{h(\theta)}\frac{\partial B}{\partial\theta}(\theta,q)]-U(\alpha)$$

To maximize revenue choose a mechanism where the participation constraint for the lowest type is binding. Then $U(\alpha)=0$.

Therefore

$$\mathrm{E}[r(\theta)]=\mathrm{E}[R_D^V(\theta,q)] \qquad (0.3)$$

where

$$R_D^V(\theta,q)=B(\theta,q)-\frac{1}{h(\theta)}\frac{\partial B}{\partial\theta}(\theta,q) \qquad (0.4)$$

(c) 2 points

$$p(\theta,q)=\theta-q$$

$$R_D^V(\theta,q)=(2+\theta)q-\tfrac{1}{2}q^2-\frac{1}{h(\theta)}q=(2+\theta-\frac{1}{h(\theta)})q-\tfrac{1}{2}q^2.$$

For the uniform case $\dfrac{1}{h(\theta)}=\dfrac{1-F(\theta)}{f(\theta)}=1-\theta$. Therefore

$$R_D^V(\theta,q)=(1+2\theta)q-\tfrac{1}{2}q^2.$$

$$MR_D^V(\theta,q)=1+2\theta-q.$$

The level set for marginal revenue is depicted below. Note that $(0,0)$ is on the level set

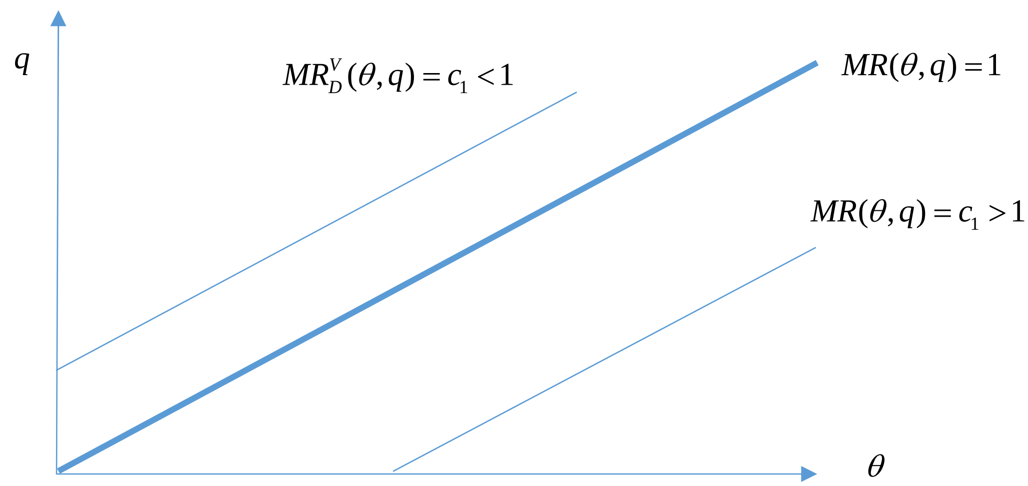


Figure 6-1: Level sets for virtual marginal revenue

$$MR_D^V(\theta,q)=1$$

Point wise maximization.

$$MR_D^V(\theta,q)=1+\theta-q=c_1=1$$

so point-wise maximization yields

$$q_D(\theta)=2\theta$$

This is increasing. Therefore it is IC and so is maximizing for the designer.

(d) 2 points

Let $R(q)$ be the cost of a q-pack. Then

$$r(\theta)=R(q(\theta))$$

And so

$$r'(\theta)=R'(q)q'(\theta)$$

Therefore

$$R'(q)=\frac{r'(\theta)}{q'(\theta)}$$

Appealing to the FOC

$$R'(q)=\frac{r'(\theta)}{q'(\theta)}=p(\theta,q(\theta))=2+\theta-q(\theta)$$

Since $q=2\theta$, $\theta=\frac{1}{2}q$

$$R'(q)=2-\tfrac{1}{2}q$$

$$R(q)=2q-\tfrac{1}{4}q^2+k$$

Since $(q(0),r(0))=(0,0)$ it follows that $k=0$.

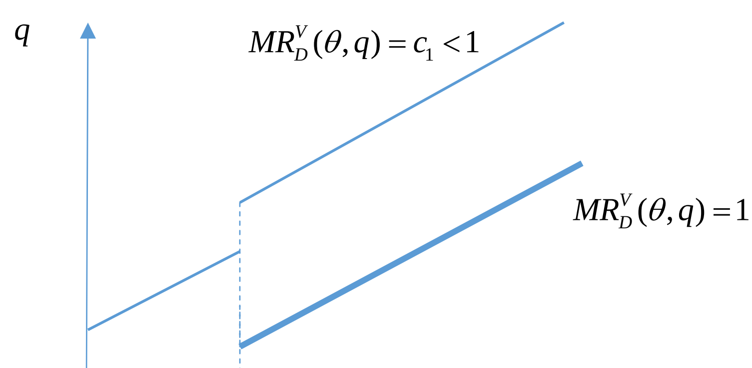
(e) 2 points

With $b>\frac{1}{2}$

$$h(\theta)=\frac{1}{2b-\theta}<\frac{1}{1-\theta},\ \ \theta<b$$

$$h(\theta)=\frac{1}{1-\theta},\ \ \theta\geq b$$

Thus $h(\theta)$ is increasing with a discontinuity at $\theta=b$. The level set for Virtual marginal revenue is depicted below.



Implementation as price a q -pack $R(q)$. The optimal $q(\theta)$ has an upward discontinuity at $q(b)$. Types are separated so $r(\theta) = R(q(\theta))$ must also have an upward discontinuity.

This is depicted below. The level set for type b touches the function $R(q)$.

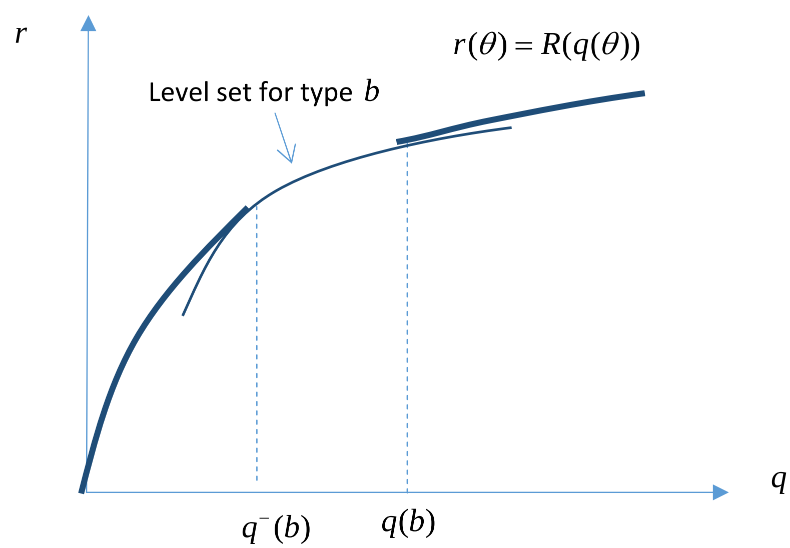


Figure 6-3 Implementation as a q -pack