UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2017)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Equilibrium with Uncertainty

Consumer $i = 1, ..., I$ has a utility function $u_i(x) = \sum_{\ell=1}^{L} \pi_\ell \log(a_i + x_{i,\ell})$, where a_i and π_ℓ are parameters and satisfy $a_i \geq 0$, $\pi_\ell > 0$ and $\sum_{\ell=1}^{L} \pi_\ell = 1$. Consumer i's endowment vector is $e_i \in R_{++}^L$.

(a) Derive consumer i's demand function $x_i(p, e_i)$ as a function of price vector and i's endowments (assume an interior solution).

(b) Show that $\sum_{i=1}^{I} x_i(p, e_i)$ depends only on p and $\sum_{i=1}^{I} e_i$ (as long as all individual demands are interior), hence there exists a representative consumer with demand function $x\left(p, \sum_{i=1}^{I} e_i\right) = \sum_{i=1}^{I} x_i(p, e_i)$.

(c) Solve for the Walrasian equilibrium when there are only two goods and two consumers with $a_1 = 4$, $a_2 = 6$, $\pi_1 = \pi_2 = \frac{1}{2}$, and $e_1 = e_2 = (20, 45)$.

For the rest of the questions, continue to assume $I = L = 2$, $\pi_1 = \pi_2 = \frac{1}{2}$, and $e_1 = e_2 = (20, 45)$, and interpret a unit of good $s \in \{1, 2\}$ as a claim to one coconut in rainfall state s (Arrow-Debreu security). Consumer i's utility is interpreted as expected utility, where π_s is the probability of state s. So Coconut is the only commodity in this economy and each consumer is also a plantation owner facing uncertainty.

(d) Suppose that the only assets they can trade ex ante are the shares of their plantations (in real term). For example, consumer i can sell the right to claim 10% of i's coconut in any state. Is it possible to achieve the Arrow-Debreu equilibrium outcome (in (c)) just by exchanging such shares? First consider the case with $a_1 = a_2 = 0$. Then consider the case with $a_1 = 4$, $a_2 = 6$ as in (c).

(e) Suppose that a riskless bond (that pays 1 coconut in every state) is available in addition to the shares. Discuss if the Arrow-Debreu outcome can be achieved in this case.

2. Simple Optimal Portfolio Problem

Anne is about to invest a fraction $\alpha \in [0,1]$ of her wealth W to some risky project with return $\widetilde{r}$, which is distributed on $[0.5, 1.5]$. The remaining amount $(1-\alpha)W$ is invested to a safe asset with return 1. So she solves the following optimization problem.

$$\max_{\alpha \in [0,1]} u(\alpha W \widetilde{r} + (1-\alpha)W).$$

where her Bernoulli utility function u satisfies $u' > 0$ on $[0.5W, 1.5W]$. Let α^* be the optimal solution for this problem.

(a) Explain why she is willing to invest at least some amount (i.e. $\alpha^* > 0$) to the risky project when the expected return of the project is higher than 1.

(b) Suppose that her Bernoulli utility function is given by $u(x) = -(x - 2W)^2$. Solve for the optimal α^* in terms of the expected return $E[\widetilde{r}]$ and the variance $VAR(\widetilde{r}) = E\left[(\widetilde{r} - E[\widetilde{r}])^2\right]$.

(c) For (b), notice that α^* is independent of W. When is this the case more generally? Suggest a class of u that has this property and explain why briefly.

(d) Consider another risky project with random return $\widetilde{R}$, which has the same expected return (i.e. $E[\widetilde{R}] = E[\widetilde{r}]$) and is second order stochastically dominated by $\widetilde{r}$. Consider the same optimal portfolio problem with this risky project and the safe asset. We still assume the quadratic Bernoulli utility function in (b). Would Anne invest more or less relative to (b)? Discuss.

3. Hard Information

A seller is trying to sell his car to a competitive market of buyers. The car's value θ is uniformly distributed on $[0, 1]$. The seller knows the θ and the buyer does not, but the seller can choose to reveal θ to the buyer. Formally, first the seller (with car θ) sends a signal $s \in \{\varnothing, \theta\}$ to the market; that is, the seller can either truthfully reveal θ or withhold the information by sending signal $\varnothing$ but he cannot lie about θ. Then the buyers bid up the price of the car to its expected value $p = E[\theta|s]$. The seller chooses his signal to maximize his revenue $u_S = p$.

(a) Assume that all types $\theta > \theta^*$ for some threshold $\theta^* \in (0, 1)$ reveal their value, $s = \theta$, while all types $\theta \leq \theta^*$ withhold their value, $s = \varnothing$. What's the price $p = E[\theta|\varnothing]$ for a car of withheld value?

(b) Characterize the Nash equilibrium (in threshold strategies) of this game. Which seller types θ reveal their type in this equilibrium?

(c) Now assume that even when the seller chooses to reveal θ, the buyer receives signal $s = \varnothing$ with probability $\alpha = 1/9$ (If the seller chooses to withhold the signal, the buyer receives signal $\varnothing$ with certainty). How do your answers to parts (a) and (b) change? (Apply Bayes' rule to calculate the expected type conditional on $s = \varnothing$, bearing in mind that this signal could have come from types $\theta \geq \theta^*$ who meant to reveal their signal, or from types $\theta \leq \theta^*$ who chose to withhold it).

(d) How does the seller's equilibrium utility in part (c) compare to his utility in part (b)?

(e) Now assume that the probability α of an erroneous $\varnothing$ signal rises. Argue qualitatively how the cutoff θ^* changes, and how the seller's equilibrium utility changes.

4 Trading in a Circle

Consider a game with N players sitting in a circle. At first, nature selects a card for each player with an integer number between 1 and M. The random process is independent for each player, and each card has a positive probability of occurring. Each player observes only his card, after which he decides upon one of two actions: "trade" or "not trade". All these choices are simultaneous, after which the game ends. Payoffs are as follows. Whenever a player selects "not trade", he receives the number on his card as a payoff. Whenever a player selects "trade", he receives the number on the card of the player closest to his right of those who selected "trade" (notice that this player is himself if nobody else selected "trade".)

(a) What strategies are rationalizable?

(b) What is the Bayesian Nash equilibrium?

5. Indivisible Public Good

Consider a two agent indivisible public good problem. The value to agent j is θ_j, $j = 1, 2$. Each buyer's value is continuously distributed with support $\Theta = [\alpha, \beta]$. The cost of the public good is k , where $2\alpha < k < 2\beta$. Consider the net contribution to social surplus direct V-C- G mechanism, i.e. agent j is asked to submit a value x_j.

(a) Let $q^*(\theta_1, \theta_2)$ be the probability that the public good is produced under social surplus maximization. Obtain an expression for social surplus and explain carefully why truth-telling is a BNE for the net contribution mechanism.

(b) Explain also why truth-telling is an equilibrium in dominant strategies.

(c) Consider a pair of values for which it is efficient to produce the public good. Show that the payoff to the designer is negative if (i) $\theta_1 + \alpha \leq \theta_2 + \alpha < k$ or (ii) $\theta_1 + \alpha < k < \theta_2 + \alpha$.

(d) Analyze the case in which $\theta_1 + \alpha \geq \theta_2 + \alpha > k$. Hence draw a conclusion as to the designer payoff.

(e) Suppose that the government is running a big deficit so its objective is profit maximization rather than social surplus maximization. Provide an example of a mechanism that is strictly profitable whenever the public good is produced.

6. Auction Design

One buyer

Buyer 1 places a value of θ on item A, where this is a draw from a distribution with support $X = [0, 1]$ and c.d.f. $F_A(\theta) \in \mathbb{C}^1$. Let $(q(\theta), r(\theta))$ be an incentive compatible mapping from values to win probabilities and expected payments.

(a) Show that the expected payment of buyer 1 can be expressed as follows: $E[J(\theta)\, q(\theta)] - U_1(0)$, where $J(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$.

Henceforth suppose that values of item A are uniformly distributed so that $F_A(\theta) = \theta$.

(b) If the seller simply sets a price, what price maximizes the designer expected revenue?

(c) Could the designer achieve a higher expected revenue using some other strategy? Explain carefully.

(d) Next suppose that there are two items for sale. Buyer 1 places a value θ_A on item A and θ_B on item B. Values are independently distributed. The value of Item B has support $Y = [0, \frac{1}{2}]$ and c.d.f. $F_B(\theta) = 2\theta_B$. What is the revenue-maximizing strategy of the seller?

Two buyers

Henceforth suppose that there are two buyers. Buyer 1 and buyer 2 value both items. The four values are independently distributed. Item A has support $X = [0, 1]$ and c.d.f. $F_A(\theta_A) = \theta_A$ (identically distributed for buyer 1 and 2). Item B has support $Y = [0, \frac{1}{2}]$ and c.d.f. $F_B(\theta) = 2\theta_B$.

(e) Characterize the revenue maximizing allocation. How might this allocation be implemented?