UCLA Department of Economics **Spring 2017**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider an economy with a representative household with N_t identical members. The household's preferences are given by,

$$\sum_{t=0}^{\infty} \beta^t N_t \left\{ \log c_t + A \log(1-h_t) \right\}.$$

Each member of the household is endowed with 1 unit of labor each period. The number of members evolves over time according to the law of motion, $N_{t+1} = \eta N_t$, $\eta > 1$.

Output is produced using the following technology:

$$Y_t = \gamma^t e^{z_t} K_t^{\mu} (N_t h_t)^{\phi} L_t^{1-\mu-\phi}$$

Here, $\gamma > 1$ is the gross rate of exogenous total factor productivity growth, K_t is total (not *per capita*) capital, Y_t is total output, and L_t is the total stock of land. Land is assumed to be a fixed factor; it cannot be produced and does not depreciate. To simplify with out loss of generality, assume that $L_t = 1$ for all *t*.

The variable z_t is technology shock that follows an autoregressive process, $z_{t+1} = \rho z_t + \varepsilon_{t+1}$, where ε is independently and identically distributed over time with mean 0 and standard deviation σ.

The resource constraint, assuming 100 percent depreciation of capital each period, is given by

$$N_t c_t + K_{t+1} \leq Y_t \ .$$

A. Formulate the social planning problem for this economy as a *stationary* dynamic program. Be clear about the transformation performed so that all variables are stationary.

B. Characterize the balanced growth path of this economy. That is, find expressions that determine c_t h_t and K_t along this growth path. In particular, solve explicitly for the growth rate of this set of variables.

C. Discuss how your answer to part *B* would change if $\phi = 1 - \mu$. In particular, what is the growth rate of income *per capita* in the two cases (part *A* and *B*)?

D. Suppose that a period is one quarter and suppose you are given annual growth rates for population and per capita income. You are also given values for factor shares, the average amount of time spent working and the annual capital-output ratio. Show how these facts can be used to calibrate γ, η, *A* and β.

E. Define a *recursive competitive equilibrium* for this economy assuming markets for labor, consumption goods, land rental, and capital services.

F. Add a real estate market to your equilibrium definition in part *E*. Derive an equation determining the price of land.

Part 2.

In this part you will study a version of the Diamond-Mortensen-Pissarides model applied to the housing market.

Setup. Time is discrete and the horizon infinite. There are two types of risk neutral agents, with identical discount factor $\beta \in (0,1)$: a measure one of households, and a large measure of real-estate specialists. A household can be either non-owner ("n") or owner ("o") of a house. The per-period utility of being a non-owner is normalized to zero.[1] The per-period flow utility of being owner is denoted by $x > 0$. Every period when it is an owner, the household receives a "moving shock" with probability δ. In that case, the household immediately sells its house to one real-estate specialist. The real-estate specialist then goes to a search-and-matching market to re-sell the house, incurring a marketing cost c every period until the house is sold. Likewise, the household goes to the same search-and-matching market to purchase a new house. When it finds a suitable vacant house, the household bargains over the price with the real estate specialist who owns that vacant house.

Notations. We let o be the measure of owners, $1-o$ the measure of non-owners, v the measure of vacant homes, and H the total measure of houses (vacant or owner-occupied). We let the per-period measure of matches between non-owner and a vacant house be $M(1-o, v)$. This is the matching function, which represents the time consuming process of finding a a suitable house. We assume, as in class, that M is concave, increasing, has constant returns to scale, and is such that $M(0,0) = 0$. We let $\theta = v/(1-o)$ denote the tightness, $q(\theta) = M(1-o, v)/v$ the buyer finding rate, and $\theta q(\theta)$ the house finding rate. We let V_o be the value of an owner, V_n the value of a non-owner, and R the value of a real-estate specialist who owns a vacant house. Finally, we let ϕ denote the fraction of the surplus appropriated by a household when it buys a house from a real-estate specialist.

1. What is the relationship between o, v, and H? (0.5pt)

2. Derive the equation that determines the steady-state measure of owners, o, as a function of M, δ and H? What is the impact on o of an increase in H and an increase in δ? (1pt)

[1] One interpretation is that the household rents an apartment, and that the rent is equal to the flow utility of living in the apartment.

3. Given that there is a large number of real-estate specialists, argue informally that, upon a moving shock, a household is able to sell its house at a price equal to R. What is the value of a real-estate specialist who does not own a vacant house? (0.5pt)

4. Argue that when a household buys a house from a real-estate specialist, the surplus of the transaction is $\Sigma = V_o - V_n - R$. Shows that the purchase price is $P = \phi R + (1 - \phi)(V_o - V_n)$. Interpret this formula. (1pt)

5. Write the Bellman equations for V_o, V_n, and R. (1pt)

6. Define an equilibrium. (1pt)

7. Use the Bellman equations to show that the surplus is $\Sigma = \frac{x+c}{1-\beta(1-\delta)+\beta\theta q(\theta)\phi+\beta q(\theta)(1-\phi)}$. Explain why the surplus is increasing in x and c, and why it is decreasing in δ. (1pt)

8. Show that $V_o - V_n$ and R are both decreasing in c. Explain why. (1pt)

9. Now let us endogenize the housing supply as follows. We assume that there a construction firms who can build new housing unit at cost γ per unit. New housing units are immediately sold to real-estate specialists.

   (a) Derive the equation that determines the equilibrium tightness θ. (1pt)

   (b) Show that, if ϕ is close to one, then the equilibrium tightness and housing supply are zero. Explain why. (1pt)

   (c) Derive comparative statics of tightness with respect to c, x, δ, and ϕ. Explain the effects. (1pt)

Part 3. Productive Government Expenditures

Preferences for the household are defined over a market consumption good and labor, and are given by:

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) - \phi(\frac{h_t^{1+\eta}}{1+\eta}) \right\} \tag{1}$$

The technology is given by:

$$A g_t^{1-\gamma-\theta} k_t^{\gamma} h_t^{\theta} \geq y_t = c_t + i_t + g_t \tag{2}$$

where g is government spending.

The evolution of the capital stock is given by:

$$k_{t+1} = i_t + (1-\delta)k_t \tag{3}$$

(A) Formulate this as a planner's problem (4 points)

(B) Derive the necessary conditions for an optimum (4 points)

(C) What values may η take on? Derive a formula for the Frisch elasticity of labor supply (3 points)

(D) Discuss the economic forces (incentives/trade-offs) that impact the efficient level of government spending. (3 points)

(E) Show that a competitive equilibrium with a lump sum tax levied on the household implements the social optimum (5 points)

(F) Suppose lump sum taxes are not available, but that a consumption tax is available. Derive a formula for the consumption tax that maximizes social welfare in the steady state. (5 points)

(G) Respond to the following statement as either True, False, or Uncertain, and explain your answer: (3 points)

"Because consumption taxes depress government spending relative to the case with lump sum taxes, households will choose to increase labor supply and investment as substitutes for government spending in production."